Estimating the stem volume of a tree by using section wise measured diameters

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Tree volumes are typically not measured directly. Some inventories just measure the diameter in 1.3 m height, measure or estimate the tree height and use a form factor to estimate stem volume. Those form factors are based on relations between diameter (and height, crown length, ...) and estimated tree volumes. Those tree volumes typical are estimated by using section wise measured stem diameters. Some methods to estimate tree volume using section wise diameters are compared. From this investigation the methods interpolating between the measured diameters and building the integral of those areas along the stem could be recommended. From the interpolation methods the one which are monotonic and non overshooting showed good results. Of the interpolation methods compared, the method developed by Steffen in 1990 showed the best performance.

Contents

1 Introduction

The volume of a tree could be distinguished in the parts below ground (roots) and those above ground (stem, branches, twigs, needles, leaves). Typical those volumes are not measured directly. By using laser scanners diameters along the stem could be measured. Still in most cases only the diameter in 1.3 m height (diameter at breast height – DBH) is measured. Sometimes tree height is measured in addition, sometimes it is estimated. Sometimes also the height of the crown base. With DBH, height and height of the crown base a form factor is used to calculate the stem volume. The form factor was estimated by using the relation between DBH, height and volume from trees where diameters along the stem (section wise) have been measured. The volume was estimated by assuming that the stem has the shape of a circle and calculating the stem area at the position of the measured diameter with $d^2\pi/4$ and multiplying this area with the length of the section where this diameter was in the middle. This method was e. g. described by Huber (1828), Krünitz (1781), Kästner (1766).

In order to get some information about the real shape of the stem, sometimes two cross-sectional diameters are measured. Their arithmetic mean will represent the stem area in a good way while their geometric mean would be theoretical better (Tischendorf, 1925). When diameters are measured orthogonal as stem disc radii the arithmetic mean of the basal areas of the individual radii is close to the true basal area of the stem (Siostrzonek, 1958).

Nowadays, it is possible to interpolate between the measured diameters and calculate the integral along the stem. The simplest interpolation between measured diameters will be linear, but also splines have been used since decades (see fig. 1). As the real volume is not known the different methods could not be ranked by comparing how near they come to the real volume. Instead, it is possible to leave one measured diameter away and let the interpolation method estimate the diameter on this now missing position.

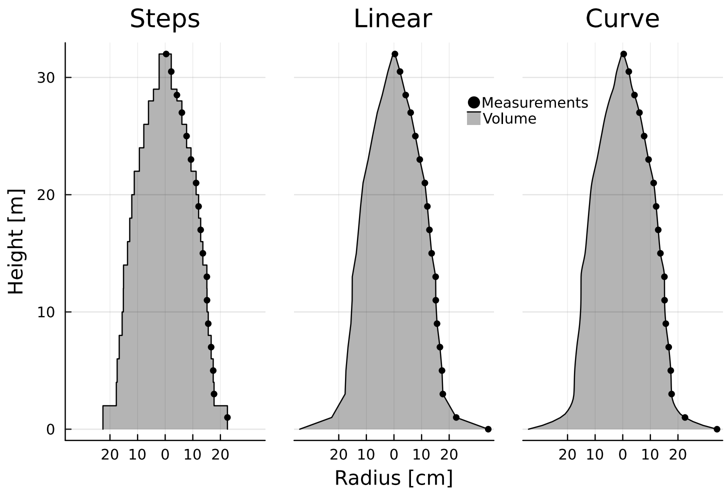

Figure 1: Different methods to estimate stem volume.

There are different definitions of stem volume. Here the stem is the part of the tree which goes from the ground continuously to the top. There is only one stem per tree. The stem includes the bark. Even if many older stems are hollow, this volume which was wood in the past but does not exist now anymore is here still counted to the stem volume. The height where a tree was or will be cut plays no role when estimating the stem volume. Here it is considered that trees are standing on flat land. If it is standing on a slope the tree height is typically measured starting from the slope side, but this point might move over time e. g. when the diameter is growing. So the point should be there where the root and the shoot come together when the tree was a seedling what will be approximately at the height of half of the stem.

2 Data

The dataset provided by Didion et al. (2024a,b) has been used. The dataset contains 15 684 measured spruce stems (Picea abies) which have been selected. The DBH ranges form 0.6 cm to 100.8 cm, height ranges form 1.6 m to 48.6 m. Most observations are in the range of DBH 10–50 cm and height 5–35 m (see fig. 2 and 3).

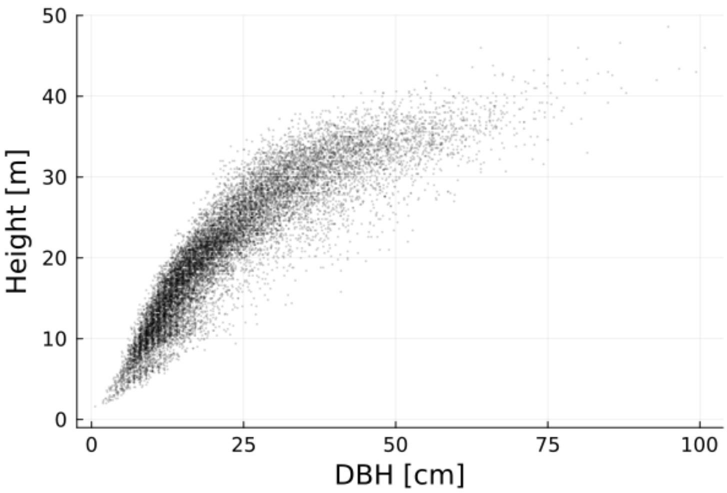

Figure 2: DBH and Height scatter plot of measured spruce.

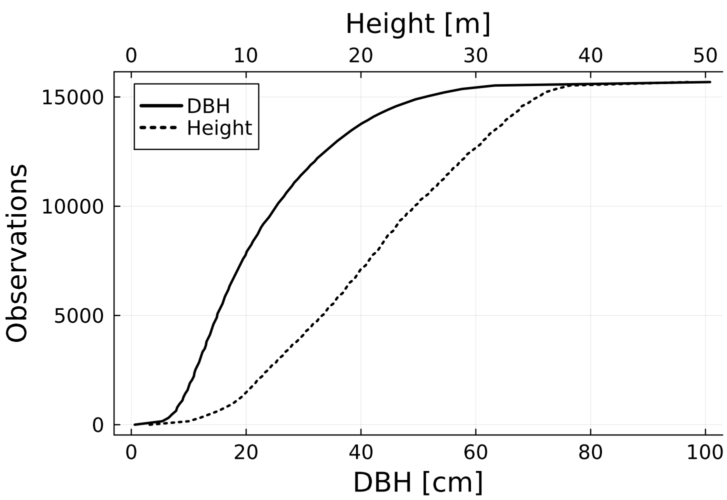

Figure 3: Cumulative observations along DBH and Height.

The stem diameter was measured starting at the height of 1 m in sections of 2 m as long as the diameter was larger than 7 cm 1 m above the measurement position. If there was a final segment larger than 7 cm but shorter than 2 m the diameter of this segment was measure at half of its length. The diameter of the final segment smaller than 7 cm was measured in its middle. In addition, the DBH (Diameter at Breast Height) at 1.3 m height and for some trees the diameter at 0.65 m height was measured. The number of measured diameters is given in tab. 1.

h	0.65	1	1.3	3	5	7	9
n	2964	15 177	15 684	14 651	13 930	13 016	12 026
h	11	13	15	17	19	21	23
n	11 037	10 025	8 939	7 812	6 634	5 604	4 617
h	25	27	29	31	33	35	37
n	3 649	2 748	1 886	1 045	475	164	58
h	39	41	43	45	Coarse	d=7	DTop
n	24	8	4	1	10 063	15 417	13 079

Table 1: Measured diameters at different tree heights.
h .. Height im [m]
n .. Number of measured diameters.
Coarse .. Diameter of the last coarse segment of the stem
d=7 .. Height where diameter is 7 cm
DTop .. Diameter of stem top with a dimater lower than 7 cm

3 Methods

As the traditional method needs the diameter in the middle of each segment, those have been measured. For methods which are interpolating between the measurements at least the diameters at the ground and at the tree top are needed in addition. As the tree top does not have a diameter of 0 cm an assumption needs to be made how wide the diameter at the tree top is. For the used data it is not relevant how large the diameter for small trees is as they are not present there. Anyhow, it was assumed that the diameter is 0.1 cm at height 0 m and is increasing up to 0.7 cm following the relation: $d_t = 0.1 + 0.6 \cdot \tanh(15 \cdot h^{1.7})$ where the diameter at the tree top (d_t) is in cm and the tree height (h) is in m (see fig. 4).

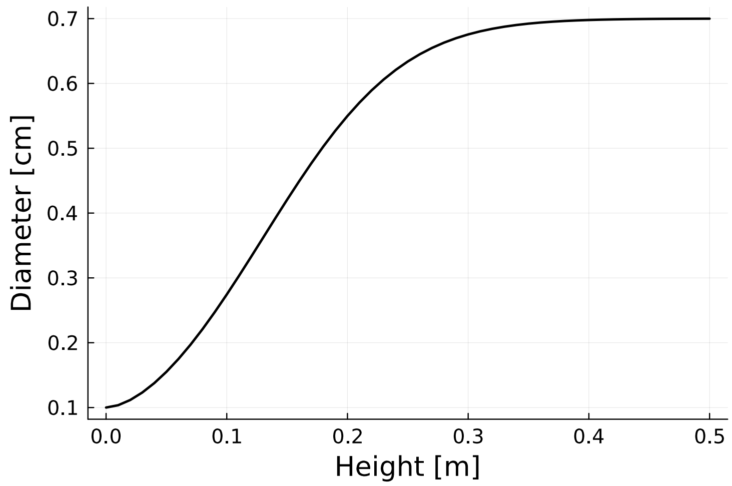

Figure 4: Assumed dependence between tree height and diameter at the top of the tree.

The tree diameter at the ground was estimated creating a regression of the form $d = c_0 + c_1(x - h)^{c_2}$ where d is the diameter at height h. c_0, c_1, c_2 are coefficients which are estimated for each tree individually and x is the highest used height. For x a height of 5 m was selected and all measured diameters until the height of 5 m have been used to estimate the regression coefficients. By using this function the diameter at height 0 could be estimated. For some interpolation methods it is useful to have an addition point behind the last point of the interpolation range. So a hypothetical diameter at height -1 m was also estimated with this equation. For 2 828 trees d_0 and d_{-1m} could be estimated by using the measured diameters up to a height of 5 m. 40 tree individual estimated d_0 where larger than 2·DBH and 412 d_{-1m} where larger than 3·DBH and have been replaced with an estimate based only on DBH and tree height. For those and the remaining trees where d_0 and d_{-1m} could not be estimated on a tree individual basis it was estimated with eq. 2 and 3.

$$d_a = \begin{cases} DBH + 1.3 & \text{if } h \geq 1.3\,m \\ h + d_t & \text{otherwise} \end{cases} \tag{1}$$

$$d_0 = d_a \cdot (1 + e^{c_0 + c_1 \cdot \ln(h) + c_2 \cdot d_a}) \tag{2}$$

$$d_{-1m} = d_a \cdot (1 + e^{c_3 + c_4 \cdot \ln(h) + c_5 \cdot d_a}) \tag{3}$$

Where d_a is an auxiliary diameter, DBH diameter in 1.3 m height in cm, h the tree height in m, d_t the diameter at the tree top, d_0 diameter in 0 m height, d_{-1m} hypothetical diameter 1 m below ground and c_0 to c_5 coefficients.

For each tree, one measured diameter was omitted, and for this position, a diameter was calculated using various interpolation methods. This was done for each single measured diameter as long as interpolation was possible for this position. Interpolation is not possible for the first and last observation. The estimated diameter was compared with the observed. The interpolation methods which have been compared are:

A Linear
B B-Spline second, third and fourth order
C Finite Difference (cubic interpolation, no tension parameter)
D Cubic cardinal splines, with tension parameter 0.1, 0.5, 0.9 and 1
E Fritsch Carlson (Fritsch and Carlson, 1980)
F Fritsch Butland (Fritsch and Butland, 1984)
G Steffen (Steffen, 1990)
H Akima (Akima, 1970)
I cubic spline keeping first and second derivatives continuous
J Hyman89 (Dougherty et al., 1989)
K C2MP – adjust first derivatives according to the simplest monotonicity preserving scheme (Huynh, 1993)
L C2MP2 – adjust first derivatives according to the extended monotonicity preserving scheme (Huynh, 1993)
M Huyn-Rational – the first derivatives are estimated through a rational function which guarantees monotonicity (Huynh, 1993)
N Van-Albada – Van Albada type of approximation for the first derivatives (Huynh, 1993)
O Van-Leer – Van Leer type of approximation for the first derivatives (Huynh, 1993)
P Brodlie – similar to Fritch Butland but takes into account the non-uniformity of the knots
Q Hyman Non Negative
R Bessel
S Fukasawa
T Quadratic Lagrange Interpolation

After finding a good method to interpolate between measured diameters the estimated stem volumes using

- Traditional sectional step method using the diameter in the middle of the section
- Linear interpolation
- Cubic spline
- Selected interpolation method

are calculated. Differences of the volumes between those methods will be shown.

All calculations have been made with the programming language Julia 1.11.3 (Bezanson et al., 2017). The interpolations have been done with the packages Interpolations v0.15.1 (Kittisopikul et al., 2024), Dierckx v0.5.4 (Dierckx, 1993) and PPInterpolation v0.7.3 (Jherek, 2024). Integrals are calculated using the package Integrals v4.5.0 (SciML, 2024) with the method QuadGKJL (Laurie, 1997). Regressions are estimated with the package GLM v1.9.0 (Julia GLM, 2025) and nonlinear with LsqFit v0.15.0 (Julia LsqFit, 2025). For data handling DataFrames v1.7.0 (Bouchet-Valat and Kamiński, 2023) and CSV v0.10.15 (Julia CSV, 2025) have been used. For plotting and formatting Plots v1.40.9 (Christ et al., 2023), LaTeXStrings v1.4.0 (Julia LaTeXStrings, 2024), Format v1.3.7 (Julia Format, 2024) and for trend lines Loess v0.7.2 (Julia Loess, 2024) have been used. For statistics StatsBase v0.34.4 (Julia StatsBase, 2024) and Statistics 1.11.3 (Bezanson et al., 2017) have been used.

4 Results

To estimate the diameter in 0 m height (d_0) using eq. 2 and the hypothetical diameter 1 m below ground (d_{-1m}) using eq. 3 the coefficients c_0 to c_5 have been estimate as:

	Coef.	Std. Error	Lower 95%	Upper 95%
c_0	−5.871 66	0.306 584	−6.472 55	−5.270 77
c_1	1.483 49	0.100 722	1.286 08	1.680 90
c_2	−0.008 991	0.001 367	−0.011 67	−0.006 31
c_3	−5.390 29	0.321 219	−6.019 86	−4.760 71
c_4	1.643 76	0.106 548	1.434 93	1.852 59
c_5	−0.012 548	0.001 531	−0.015 55	−0.009 54

The residuals along prediction, d_a, tree height and height / d_a are shown for d_0 in fig. 5 and for d_{-1m} in fig. 6 which show no trend.

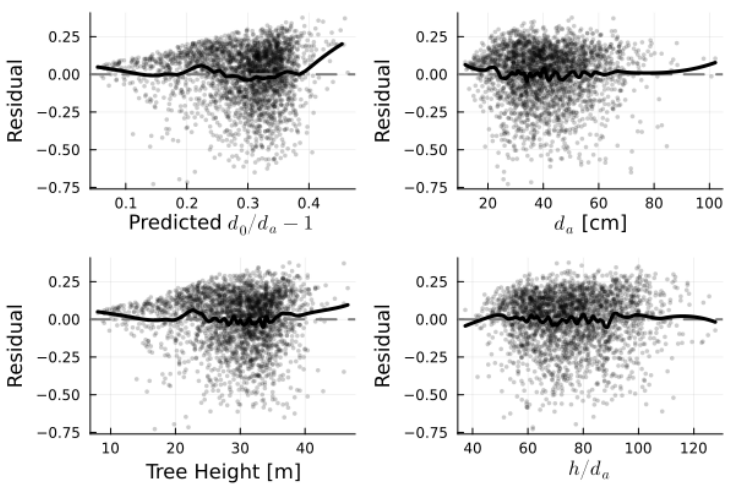

Figure 5: Residual plots of d_0 model.
Residual .. (predicted d_0 - "observed" d_0)$/d_a - 1$

The number of observations between measured and estimated diameters at different tree heights is given in tab. 2.

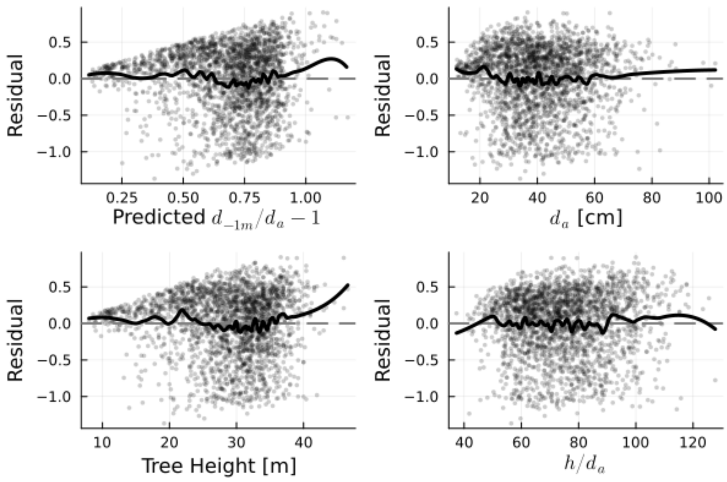

Figure 6: Residual plots of d_{-1m} model.
Residual .. (predicted d_{-1m} - "observed" d_{-1m})$/d_a - 1$

Regular measured Coarse diameters higher than 27 m are not shown as the number of observations is declining, and most methods show the same behavior as shown at the lower diameters. The last segment of coarse wood, the diameter of 7 cm and the diameter of the top are also given.

The median between the estimated and the observed diameters for different heights and methods are shown in tab. 3. The linear method (A) is overestimating in the concave sections on the stem basis and overestimating in the convex sections. Many methods find quite good estimates in the higher stem sections and show median differences lower than 0.03 cm. Really strong in this region is Method G having 6 times the lowest median difference. In addition, it has the lowest median difference at DTop and in the sections of lower height the differences reach a maximum of 0.18 cm. Methods I, J, K, Q and R belong 4 times to the methods of the lowest differences. Method E has with −0.09 cm the lowest maximum median difference, but it underestimates most of the time. Up to a height of 3 m method E has 2 times the lowest difference.

As the arithmetic mean could be strong influenced by outliers it was calculated using only values which are between the 2.5 % and 97.5 % quantile. It is shown in tab. 4. G, J and K belong 10 times and B2, B3, B4, I and Q belong 9 times to the methods with the lowest differences. Methods J and K have with 0.09 cm the lowest maximum difference. Up to a height of 3 m method G has 2 times the lowest difference.

Like for the arithmetic mean also for calculating the standard deviation only values which are between the 2.5 % and 97.5 % quantile have been used. The results are shown in tab. 5. The lowest deviations show the linear interpolation (A) with 13 times having the lowest value followed by D1 with 12 and D0.9 with 11 times. Methods D1 has with 0.80 cm the lowest maximum deviance. A similar result is given by comparing the distance between 2.5 % and 97.5% quantile (tab. 6) and between 25 % and 75 % quantile (tab. 7).

By combining the results showing the mean difference and the deviation a method should be found which has a low bias and also low deviation. From the bias the ranking of the methods would be G, J, K, I, Q and R. Out of those methods G also shows good performance according to the deviation. Overall the method of Steffen (1990) (G) looks like to be recommendable to be used to interpolate between measured diameters and integrate those to a stem volume.

Fig. 7 shows the relations between volumes estimated by sectional step method using the diameter in the middle of the section (Section), linear interpolation (Linear) and cubic spline (Spline) and the volume estimated by interpolating using the method by Steffen (1990). The volume by section wise calculation is typically lower than Steffen. Especially for small trees this difference could be in the range of 20 % of the volume. The linear interpolation is underestimating small trees and overestimating marginal for larger trees. The spine interpolation is over a wide range consistent with Steffen but has sometimes very large differences.

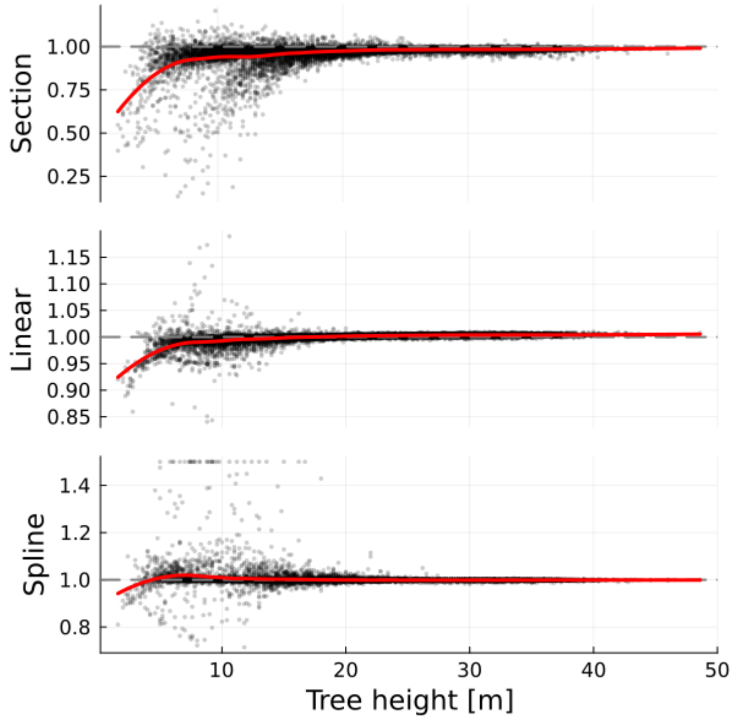

Figure 7: Relation of different volume estimates along tree height.
Section .. Section wise Volume / Steffen Volume, Linear .. Volume by linear interpolation / Steffen Volume, Spline .. Volume by interpolation with cubic spline / Steffen Volume. Relations larger than 1.5 are shown as 1.5

Fig. 8 compares the result of different interpolation methods. If there are many measurements with a continuous pattern the methods differ marginal (TreeID: 3). If there are only some observations, but the pattern of the measurements looks simple the differences between the methods are still small (TreeID: 25). In case of an abrupt break of the trend and missing observations over wide height range the differences are getting large (TreeID: 17314, 8402). Interpolation methods like splines could exceed the range of values between two successive points, what might be possible in reality but could lead to unrealistic values like negative diameters (TreeID: 17314). Non overshooting methods like Steffen do not exceed the range of values between two successive points and look like to estimate robust results.

5 Discussion

Müller (1902, 1923) report a systematic error of 3 % when using the section wise volume calculation with the cross-sectional area in the middle of the stem section. Prodan

h	0.65	1	1.3	3	5	7	9	11	13	15	17	19	21	23	25	27	Coarse	d=7	DTop
n	2 960	2 960	14 527	13 807	12 895	11 909	10 940	9 959	8 890	7 781	6 616	5 596	4 615	3 647	2 746	1 884	9 824	7 986	12 519

Table 2: Number of interpolated diameters at different tree heights.

h	0.65	1	1.3	3	5	7	9	11	13	15	17	19	21	23	25	27	Coarse	d=7	DTop
A	0.51	0.39	0.30	0.31	**0.00**	−0.05	−0.10	−0.10	−0.10	−0.15	−0.15	−0.15	−0.20	−0.20	−0.25	−0.25	**0.00**	−0.14	−0.15
B2	−0.16	0.13	−0.33	−0.36	−0.01	**0.00**	0.01	0.01	−0.01	0.01	0.01	−0.01	**0.01**	0.01	−0.01	−0.03	0.06	−0.09	0.14
B3	−0.16	0.12	−0.29	−0.30	0.04	−0.02	0.01	**0.00**	−0.01	0.01	0.01	−0.01	**0.01**	0.01	**0.00**	−0.03	0.05	−0.09	0.27
B4	−0.18	0.12	−0.31	0.77	−0.14	0.06	−0.01	**0.00**	−0.01	**0.00**	0.02	−0.01	**0.01**	0.01	0.01	−0.02	0.05	−0.08	0.33
C	−0.07	0.10	−0.14	−0.09	−0.11	−0.03	−0.01	−0.02	−0.04	−0.03	−0.02	−0.05	−0.04	−0.05	−0.09	−0.08	0.06	−0.09	−0.07
D0.1	0.07	0.05	0.07	0.24	−0.07	−0.04	−0.05	−0.05	−0.07	−0.07	−0.07	−0.10	−0.10	−0.12	−0.16	−0.15	0.03	−0.08	−0.04
D0.5	**0.03**	0.17	0.25	0.29	−0.05	−0.05	−0.06	−0.07	−0.09	−0.10	−0.10	−0.13	−0.14	−0.17	−0.20	−0.19	−0.08	**0.03**	0.26
D0.9	−0.04	0.29	0.45	0.34	−0.02	−0.05	−0.09	−0.09	−0.10	−0.13	−0.13	−0.15	−0.19	−0.20	−0.24	−0.24	−0.20	0.14	0.56
D1	−0.05	0.32	0.50	0.36	**0.00**	−0.05	−0.10	−0.10	−0.10	−0.15	−0.15	−0.15	−0.20	−0.20	−0.25	−0.25	−0.23	0.16	0.64
E	−0.08	0.08	**−0.03**	**−0.03**	−0.09	−0.02	−0.01	−0.02	−0.04	−0.03	−0.01	−0.05	−0.04	−0.05	−0.09	−0.08	0.06	−0.09	−0.06
F	−0.08	0.07	0.12	0.08	−0.06	−0.02	−0.01	−0.01	−0.03	−0.02	−0.01	−0.04	−0.03	−0.04	−0.06	−0.07	0.05	−0.08	−0.02
G	−0.18	0.05	0.08	0.08	−0.08	−0.02	**0.00**	**0.00**	**0.00**	0.01	0.02	**0.00**	**0.01**	**0.00**	−0.02	−0.03	0.06	−0.08	**−0.01**
H	−0.06	0.07	0.07	0.14	−0.02	−0.03	−0.02	−0.02	−0.03	−0.03	−0.03	−0.06	−0.05	−0.06	−0.09	−0.09	0.05	−0.09	0.05
I	−0.23	0.13	−0.30	−0.29	0.03	−0.02	0.01	**0.00**	−0.01	**0.00**	0.01	−0.01	**0.01**	0.01	**0.00**	−0.03	0.06	−0.08	0.05
J	−0.23	0.11	−0.10	−0.15	0.02	−0.02	0.01	**0.00**	−0.01	**0.00**	0.01	−0.01	**0.01**	0.01	**0.00**	−0.03	0.06	−0.08	0.05
K	−0.23	0.11	−0.10	−0.15	0.02	−0.02	0.01	**0.00**	−0.01	**0.00**	0.01	−0.01	**0.01**	0.01	**0.00**	−0.03	0.06	−0.08	0.05
L	−0.14	**0.01**	−0.10	−0.10	−0.14	−0.02	**0.00**	**0.00**	−0.02	−0.01	**0.00**	−0.03	−0.02	−0.04	−0.05	−0.06	0.11	−0.13	−0.11
M	−0.06	0.06	0.10	0.11	−0.07	−0.03	−0.02	−0.02	−0.04	−0.03	−0.02	−0.05	−0.04	−0.06	−0.08	−0.09	0.05	−0.09	**0.01**
N	**−0.03**	0.06	0.25	0.27	−0.03	−0.03	−0.02	−0.03	−0.04	−0.03	−0.03	−0.06	−0.05	−0.07	−0.08	−0.09	0.04	−0.08	0.07
O	−0.06	0.06	0.15	0.17	−0.05	−0.03	−0.02	−0.02	−0.04	−0.03	−0.02	−0.05	−0.04	−0.06	−0.08	−0.09	0.05	−0.09	0.03
P	−0.07	0.07	0.12	0.08	−0.06	−0.02	−0.01	−0.01	−0.03	−0.02	−0.01	−0.04	−0.03	−0.04	−0.06	−0.07	0.05	−0.08	0.04
Q	−0.23	0.13	−0.30	−0.29	0.03	−0.02	0.01	**0.00**	−0.01	**0.00**	0.01	−0.01	**0.01**	0.01	**0.00**	−0.03	0.06	−0.08	0.05
R	−0.18	0.17	−0.30	−0.40	−0.14	−0.02	**0.00**	0.01	**0.00**	0.01	0.02	**0.00**	0.02	**0.00**	−0.02	−0.02	0.06	−0.08	0.06
S	−0.05	0.07	0.11	0.11	−0.08	−0.03	−0.02	−0.02	−0.04	−0.03	−0.02	−0.05	−0.05	−0.06	−0.09	−0.09	0.05	−0.09	**0.01**
T	−0.14	0.22	−0.39	−0.95	−0.31	−0.03	−0.03	−0.03	−0.03	**0.00**	**0.00**	−0.03	−0.03	−0.03	−0.03	**0.00**	0.07	−0.09	0.13

Table 3: Median difference [cm] between estimated - observed diameter.

h	0.65	1	1.3	3	5	7	9	11	13	15	17	19	21	23	25	27	Coarse	d=7	DTop
A	0.78	0.48	0.49	0.54	0.01	−0.07	−0.09	−0.10	−0.12	−0.13	−0.15	−0.17	−0.19	−0.24	−0.26	−0.26	−0.02	−0.13	−0.15
B2	−0.07	0.14	−0.30	−0.62	−0.01	−0.01	**0.00**	0.01	**0.00**	**0.00**	**0.01**	**0.00**	**0.01**	**0.00**	**0.00**	**−0.02**	0.05	−0.07	0.09
B3	−0.08	0.12	−0.24	−0.55	0.07	−0.03	0.01	**0.00**	**0.00**	**0.00**	**0.01**	**0.00**	**0.01**	**0.00**	**0.00**	**−0.02**	0.05	−0.07	0.13
B4	−0.17	0.12	−0.27	0.54	0.04	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.01**	**0.00**	**0.01**	0.01	0.02	**−0.02**	0.05	−0.07	0.09
C	0.16	0.08	−0.06	−0.10	−0.14	−0.03	−0.02	−0.02	−0.03	−0.03	−0.03	−0.04	−0.05	−0.07	−0.07	−0.08	0.04	−0.08	−0.08
D0.1	0.33	**0.02**	0.22	0.43	−0.07	−0.05	−0.05	−0.06	−0.07	−0.08	−0.08	−0.10	−0.11	−0.14	−0.15	−0.16	**0.01**	−0.06	−0.05
D0.5	0.21	0.18	0.45	0.50	−0.04	−0.06	−0.07	−0.08	−0.09	−0.10	−0.11	−0.13	−0.15	−0.18	−0.20	−0.21	−0.12	**0.05**	0.25
D0.9	0.08	0.35	0.69	0.58	**0.00**	−0.07	−0.08	−0.10	−0.12	−0.13	−0.14	−0.16	−0.18	−0.23	−0.25	−0.25	−0.25	0.16	0.56
D1	0.05	0.39	0.75	0.60	0.01	−0.07	−0.09	−0.10	−0.12	−0.13	−0.15	−0.17	−0.19	−0.24	−0.26	−0.26	−0.28	0.19	0.64
E	0.15	0.05	0.05	**0.04**	−0.11	−0.03	−0.02	−0.02	−0.03	−0.03	−0.03	−0.04	−0.04	−0.07	−0.07	−0.08	0.04	−0.08	−0.07
F	0.09	0.04	0.24	0.17	−0.07	−0.03	−0.01	−0.01	−0.03	−0.02	−0.02	−0.04	−0.03	−0.05	−0.05	−0.07	0.03	−0.07	−0.02
G	**−0.02**	**0.02**	0.18	0.16	−0.10	−0.02	**0.00**	**0.00**	**0.00**	**0.00**	**0.01**	**0.00**	**0.01**	−0.01	−0.01	**−0.02**	0.05	−0.07	−0.03
H	0.08	0.03	0.18	0.21	−0.01	−0.03	−0.02	−0.02	−0.04	−0.03	−0.04	−0.05	−0.05	−0.07	−0.08	−0.09	0.03	−0.08	0.03
I	−0.10	0.12	−0.25	−0.55	0.07	−0.03	0.01	**0.00**	**0.00**	**0.00**	**0.01**	**0.00**	**0.01**	**0.00**	**0.00**	**−0.02**	0.05	−0.07	0.02
J	−0.07	0.09	**−0.04**	−0.08	0.02	−0.03	0.01	**0.00**	**0.00**	**0.00**	**0.01**	**0.00**	**0.01**	**0.00**	**0.00**	**−0.02**	0.05	−0.07	0.02
K	−0.07	0.09	**−0.04**	−0.09	0.02	−0.03	0.01	**0.00**	**0.00**	**0.00**	**0.01**	**0.00**	**0.01**	**0.00**	**0.00**	**−0.02**	0.05	−0.07	0.02
L	0.10	−0.05	−0.06	−0.05	−0.17	−0.03	−0.01	−0.01	−0.02	−0.01	**−0.01**	−0.02	−0.02	−0.04	−0.05	−0.06	0.09	−0.12	−0.14
M	0.13	0.03	0.22	0.21	−0.07	−0.03	−0.02	−0.02	−0.03	−0.03	−0.03	−0.05	−0.05	−0.07	−0.07	−0.09	0.04	−0.07	0.01
N	0.14	**0.02**	0.41	0.40	−0.02	−0.03	−0.02	−0.03	−0.04	−0.04	−0.04	−0.05	−0.05	−0.08	−0.08	−0.09	0.02	−0.06	0.08
O	0.13	**0.02**	0.29	0.28	−0.05	−0.03	−0.02	−0.02	−0.04	−0.03	−0.04	−0.05	−0.05	−0.07	−0.08	−0.09	0.03	−0.07	0.03
P	0.09	0.04	0.24	0.17	−0.07	−0.03	−0.01	−0.01	−0.03	−0.02	−0.02	−0.04	−0.03	−0.05	−0.05	−0.07	0.03	−0.07	0.04
Q	−0.10	0.12	−0.25	−0.55	0.07	−0.03	0.01	**0.00**	**0.00**	**0.00**	**0.01**	**0.00**	**0.01**	**0.00**	**0.00**	**−0.02**	0.05	−0.07	0.02
R	−0.04	0.18	−0.28	−0.61	−0.20	−0.02	**0.00**	0.01	**0.00**	**0.00**	**0.01**	**0.00**	**0.01**	−0.01	−0.01	**−0.02**	0.05	−0.07	0.02
S	0.13	0.03	0.25	0.22	−0.08	−0.03	−0.02	−0.02	−0.04	−0.04	−0.04	−0.05	−0.05	−0.07	−0.08	−0.09	0.03	−0.07	**0.00**
T	−0.12	0.25	−0.39	−1.44	−0.45	−0.06	−0.02	−0.02	−0.02	−0.02	−0.02	−0.03	−0.03	−0.05	−0.03	**−0.02**	0.06	−0.07	0.08

Table 4: Mean difference [cm] between estimated - observed diameter skipping upper and lower 2.5 % of observations.

h	0.65	1	1.3	3	5	7	9	11	13	15	17	19	21	23	25	27	Coarse	d=7	DTop
A	1.16	0.62	0.61	0.74	0.29	**0.26**	**0.28**	**0.29**	**0.30**	**0.33**	**0.35**	0.38	**0.40**	**0.43**	**0.46**	**0.48**	**0.29**	**0.31**	**0.42**
B2	0.90	0.56	0.60	1.27	0.45	0.31	0.32	0.34	0.36	0.39	0.42	0.45	0.48	0.51	0.56	0.58	0.32	0.33	0.77
B3	0.89	0.56	0.59	1.32	0.59	0.35	0.34	0.35	0.38	0.41	0.44	0.47	0.50	0.54	0.59	0.62	0.31	0.34	1.68
B4	0.91	0.56	0.63	2.91	1.73	0.80	0.49	0.44	0.45	0.49	0.53	0.57	0.62	0.68	0.75	0.84	0.33	0.36	2.63
C	1.03	**0.55**	0.55	0.65	0.31	0.28	0.29	0.31	0.32	0.35	0.38	0.41	0.43	0.46	0.49	0.51	0.30	0.32	0.47
D0.1	1.09	0.56	0.59	0.64	**0.27**	0.27	**0.28**	0.30	0.31	0.34	0.36	0.39	0.42	0.44	0.47	0.49	0.30	0.32	0.44
D0.5	0.94	0.56	0.65	0.70	**0.27**	**0.26**	**0.28**	**0.29**	0.31	**0.33**	0.36	0.38	0.41	0.44	**0.46**	**0.48**	0.30	0.32	0.43
D0.9	0.82	0.59	0.73	0.75	0.29	**0.26**	**0.28**	**0.29**	**0.30**	**0.33**	**0.35**	**0.37**	**0.40**	**0.43**	**0.46**	**0.48**	0.31	0.33	0.43
D1	**0.80**	0.60	0.76	0.77	0.29	**0.26**	**0.28**	**0.29**	**0.30**	**0.33**	**0.35**	0.38	**0.40**	**0.43**	**0.46**	**0.48**	0.31	0.33	**0.42**
E	1.06	0.57	0.49	0.54	0.31	0.28	0.29	0.31	0.33	0.35	0.38	0.41	0.43	0.46	0.49	0.51	0.30	0.32	0.48
F	1.02	0.57	0.51	0.50	0.28	0.28	0.29	0.31	0.33	0.35	0.38	0.41	0.43	0.46	0.50	0.52	0.30	0.33	0.50
G	1.03	0.58	0.48	**0.48**	0.29	0.28	0.30	0.32	0.33	0.36	0.39	0.42	0.45	0.48	0.51	0.53	0.31	0.32	0.52
H	1.00	0.58	0.57	0.54	0.29	0.27	0.29	0.31	0.32	0.35	0.38	0.40	0.43	0.46	0.49	0.51	0.30	0.32	0.58
I	0.99	0.58	0.60	1.33	0.59	0.35	0.34	0.36	0.38	0.41	0.44	0.48	0.50	0.54	0.59	0.62	0.32	0.33	0.66
J	1.08	0.59	0.49	0.66	0.44	0.34	0.34	0.35	0.38	0.41	0.44	0.47	0.50	0.54	0.59	0.62	0.32	0.33	0.66
K	1.08	0.59	0.49	0.64	0.44	0.34	0.34	0.35	0.38	0.41	0.44	0.47	0.50	0.54	0.59	0.62	0.32	0.33	0.66
L	1.15	0.63	**0.45**	0.49	0.33	0.29	0.30	0.32	0.34	0.37	0.39	0.43	0.46	0.48	0.52	0.54	0.32	0.32	0.58
M	1.04	0.57	0.51	0.52	0.28	0.27	0.29	0.31	0.32	0.35	0.38	0.40	0.43	0.46	0.49	0.51	0.30	0.32	0.54
N	1.03	0.58	0.56	0.57	0.28	0.27	0.29	0.31	0.32	0.35	0.38	0.40	0.43	0.46	0.50	0.52	**0.29**	0.33	0.55
O	1.03	0.58	0.52	0.53	0.28	0.27	0.29	0.31	0.32	0.35	0.38	0.40	0.43	0.46	0.49	0.51	0.30	0.32	0.54
P	1.02	0.58	0.51	0.52	0.28	0.28	0.29	0.31	0.33	0.35	0.38	0.41	0.43	0.46	0.50	0.52	0.30	0.33	0.57
Q	0.99	0.58	0.60	1.33	0.59	0.35	0.34	0.36	0.38	0.41	0.44	0.48	0.50	0.54	0.59	0.62	0.32	0.33	0.66
R	0.96	**0.55**	0.58	1.04	0.36	0.29	0.30	0.32	0.34	0.36	0.39	0.42	0.45	0.48	0.52	0.54	0.31	0.33	0.61
S	1.03	0.58	0.54	0.55	0.28	0.27	0.29	0.31	0.32	0.35	0.38	0.40	0.43	0.46	0.49	0.51	0.30	0.32	0.52
T	0.82	0.58	0.62	1.93	0.57	0.32	0.32	0.33	0.35	0.37	0.40	0.43	0.46	0.50	0.53	0.56	0.32	0.33	0.71

Table 5: Standard deviation [cm] between estimated - observed diameter skipping upper and lower 2.5 % of observations.

h	0.65	1	1.3	3	5	7	9	11	13	15	17	19	21	23	25	27	Coarse	d=7	DTop
A	5.81	3.17	3.02	3.79	1.60	**1.40**	**1.45**	**1.45**	**1.55**	**1.65**	**1.80**	**1.90**	**2.00**	2.19	**2.27**	**2.35**	**1.47**	**1.59**	**1.90**
B2	4.89	2.93	3.58	7.24	2.61	1.69	1.69	1.77	1.86	1.97	2.13	2.29	2.37	2.55	2.80	2.88	1.67	1.70	3.79
B3	4.81	2.97	3.52	7.62	3.46	1.92	1.79	1.86	1.94	2.07	2.24	2.40	2.50	2.65	2.93	3.12	1.65	1.71	9.37
B4	4.84	3.00	3.83	19.01	10.82	4.85	2.71	2.27	2.33	2.52	2.76	2.97	3.08	3.31	3.63	3.98	1.70	1.83	16.49
C	5.59	2.95	3.16	4.02	1.70	1.49	1.55	1.60	1.67	1.78	1.91	2.05	2.18	2.32	2.49	2.57	1.57	1.64	2.22
D0.1	5.97	2.97	3.11	3.40	**1.53**	1.44	1.51	1.55	1.58	1.72	1.84	1.96	2.07	2.22	2.40	2.42	1.54	1.64	2.05
D0.5	5.23	2.99	3.25	3.61	**1.53**	1.42	1.49	1.52	**1.55**	1.70	1.81	1.92	2.03	2.18	2.35	2.39	1.53	1.66	2.01
D0.9	4.56	3.04	3.48	3.87	1.60	1.41	1.46	1.47	**1.55**	1.66	**1.80**	**1.90**	2.01	**2.16**	2.28	2.36	1.55	1.69	1.92
D1	4.38	3.13	3.55	3.94	1.60	**1.40**	**1.45**	**1.45**	**1.55**	**1.65**	**1.80**	**1.90**	**2.00**	2.19	**2.27**	**2.35**	1.56	1.70	**1.90**
E	6.25	3.00	2.94	3.13	1.67	1.51	1.56	1.63	1.69	1.80	1.94	2.07	2.18	2.32	2.54	2.57	1.57	1.64	2.28
F	5.82	3.04	2.77	2.83	1.57	1.50	1.56	1.62	1.69	1.79	1.93	2.05	2.17	2.35	2.60	2.54	1.56	1.67	2.33
G	5.93	3.07	**2.74**	**2.72**	1.57	1.52	1.58	1.65	1.73	1.85	1.98	2.10	2.25	2.38	2.66	2.63	1.61	1.66	2.44
H	5.31	3.06	3.20	3.37	1.59	1.47	1.53	1.56	1.64	1.77	1.89	2.01	2.12	2.32	2.49	2.51	1.55	1.64	2.74
I	5.47	3.08	3.56	7.69	3.47	1.93	1.80	1.86	1.94	2.07	2.25	2.40	2.51	2.65	2.93	3.14	1.68	1.70	3.23
J	6.33	3.10	3.09	3.69	2.29	1.82	1.77	1.83	1.92	2.07	2.22	2.39	2.49	2.64	2.91	3.14	1.67	1.70	3.22
K	6.33	3.10	3.09	3.56	2.29	1.82	1.77	1.84	1.92	2.07	2.22	2.39	2.49	2.64	2.92	3.14	1.67	1.70	3.20
L	7.18	3.28	2.85	2.84	1.73	1.56	1.64	1.66	1.74	1.88	2.01	2.18	2.30	2.38	2.59	2.70	1.66	1.65	2.97
M	5.91	3.02	2.83	2.90	1.56	1.49	1.55	1.61	1.67	1.77	1.92	2.03	2.14	2.31	2.57	2.52	1.56	1.66	2.54
N	5.76	3.10	2.87	3.00	1.57	1.48	1.55	1.61	1.66	1.80	1.92	2.03	2.16	2.34	2.57	2.54	1.55	1.67	2.58
O	5.88	3.05	2.82	2.87	1.55	1.49	1.54	1.61	1.67	1.78	1.92	2.02	2.15	2.34	2.57	2.53	1.55	1.66	2.55
P	5.81	3.04	2.78	2.96	1.57	1.51	1.56	1.62	1.70	1.80	1.94	2.06	2.17	2.35	2.60	2.54	1.56	1.67	2.72
Q	5.47	3.08	3.56	7.68	3.47	1.93	1.80	1.86	1.94	2.07	2.25	2.40	2.51	2.65	2.93	3.14	1.68	1.70	3.23
R	5.28	**2.92**	3.46	5.87	1.96	1.54	1.60	1.67	1.75	1.85	2.00	2.15	2.25	2.40	2.65	2.67	1.62	1.66	2.99
S	5.86	3.05	2.84	3.01	1.54	1.48	1.53	1.60	1.66	1.77	1.92	2.01	2.14	2.31	2.57	2.52	1.56	1.65	2.45
T	**4.33**	3.04	3.71	10.17	3.02	1.73	1.70	1.77	1.80	1.93	2.13	2.24	2.32	2.53	2.67	2.77	1.70	1.70	3.52

Table 6: Range between 2.5 % and 97.5 % quantile [cm] between estimated - observed diameter.

h	0.65	1	1.3	3	5	7	9	11	13	15	17	19	21	23	25	27	Coarse	d=7	DTop
A	1.58	0.84	0.77	0.88	0.40	**0.35**	0.40	**0.40**	0.45	**0.45**	**0.50**	**0.55**	0.60	0.65	**0.60**	**0.65**	**0.38**	**0.44**	**0.60**
B2	1.11	0.72	0.69	1.55	0.57	0.42	0.44	0.48	0.52	0.55	0.60	0.65	0.69	0.74	0.80	0.86	0.43	0.47	1.13
B3	1.14	0.73	0.67	1.59	0.72	0.48	0.47	0.50	0.54	0.58	0.63	0.69	0.73	0.78	0.85	0.93	0.42	0.47	2.29
B4	1.20	0.73	0.70	3.00	1.90	0.97	0.66	0.61	0.65	0.69	0.75	0.84	0.90	0.96	1.06	1.24	0.44	0.51	3.27
C	0.98	**0.71**	0.63	0.75	0.41	0.36	0.40	0.44	0.46	0.49	0.54	0.59	0.62	0.65	0.70	0.75	0.41	0.45	0.71
D0.1	0.95	0.72	0.71	0.76	**0.36**	0.36	0.39	0.42	0.44	0.47	0.51	0.57	0.60	0.63	0.66	0.70	0.41	**0.44**	0.66
D0.5	0.97	0.73	0.81	0.83	**0.36**	**0.35**	**0.38**	0.41	**0.43**	0.46	0.51	**0.55**	**0.58**	**0.62**	0.64	0.69	0.41	**0.44**	0.64
D0.9	1.03	0.78	0.95	0.90	0.38	**0.35**	0.39	**0.40**	0.44	**0.45**	**0.50**	**0.55**	**0.58**	0.64	0.62	0.67	0.43	0.45	0.61
D1	1.03	0.81	0.99	0.92	0.40	**0.35**	0.40	**0.40**	0.45	**0.45**	**0.50**	**0.55**	0.60	0.65	**0.60**	**0.65**	0.43	0.45	**0.60**
E	0.98	0.73	0.55	0.68	0.41	0.37	0.40	0.44	0.46	0.50	0.54	0.59	0.62	0.65	0.70	0.75	0.41	0.45	0.73
F	1.11	0.75	0.61	0.62	0.38	0.37	0.40	0.43	0.46	0.50	0.54	0.59	0.62	0.67	0.70	0.75	0.40	0.46	0.76
G	1.07	0.76	0.57	0.62	0.40	0.39	0.40	0.45	0.47	0.52	0.55	0.60	0.65	0.69	0.73	0.78	0.42	0.46	0.80
H	1.20	0.76	0.68	**0.61**	0.38	0.36	0.40	0.44	0.46	0.49	0.54	0.59	0.63	0.66	0.70	0.73	0.41	0.46	0.86
I	1.15	0.76	0.68	1.61	0.72	0.48	0.47	0.50	0.54	0.58	0.63	0.69	0.73	0.78	0.85	0.94	0.43	0.47	0.97
J	1.15	0.77	0.53	0.90	0.61	0.47	0.47	0.50	0.54	0.58	0.63	0.69	0.73	0.78	0.85	0.94	0.43	0.47	0.98
K	1.15	0.77	0.53	0.89	0.61	0.47	0.47	0.50	0.54	0.58	0.63	0.69	0.73	0.78	0.85	0.94	0.43	0.47	0.97
L	**0.80**	0.86	**0.51**	0.65	0.45	0.40	0.42	0.45	0.47	0.52	0.57	0.61	0.65	0.70	0.75	0.75	0.45	0.45	0.83
M	1.07	0.75	0.62	0.63	0.38	0.37	0.40	0.43	0.46	0.49	0.53	0.58	0.62	0.66	0.69	0.74	0.41	0.45	0.81
N	1.17	0.77	0.70	0.71	0.37	0.37	0.40	0.43	0.46	0.49	0.53	0.58	0.62	0.66	0.69	0.74	0.40	0.46	0.83
O	1.09	0.76	0.64	0.65	0.37	0.37	0.40	0.43	0.46	0.49	0.53	0.58	0.62	0.66	0.69	0.74	0.40	0.45	0.82
P	1.10	0.75	0.61	0.63	0.38	0.37	0.40	0.43	0.46	0.50	0.54	0.59	0.62	0.67	0.70	0.75	0.40	0.46	0.85
Q	1.15	0.76	0.68	1.61	0.72	0.48	0.47	0.50	0.54	0.58	0.63	0.69	0.73	0.78	0.85	0.94	0.43	0.47	0.97
R	1.07	**0.71**	0.67	1.27	0.46	0.40	0.40	0.45	0.47	0.52	0.57	0.60	0.65	0.70	0.73	0.78	0.42	0.46	0.91
S	1.09	0.75	0.67	0.67	0.38	0.37	0.40	0.43	0.46	0.49	0.53	0.59	0.62	0.66	0.69	0.73	0.41	0.45	0.80
T	1.07	0.75	0.72	2.45	0.70	0.43	0.43	0.43	0.50	0.50	0.57	0.60	0.67	0.70	0.73	0.80	0.43	0.46	1.04

Table 7: Inter quantile range between 25 % and 75 % quantile [cm] between estimated - observed diameter.

(1965) comes to the conclusion that the volume of concave trees is underestimated and of convex overestimated with this method. Using linear interpolation the volume of concave sections is overestimated, those of convex underestimated. Monotonic non overshooting interpolations showed good results for estimating stem shape and with this tree volumes. From those which have been compared here the method of Steffen (1990) showed good performance.

Extrapolation is uncertain. Measurements at or close to the beginning and the end of the stem could avoid the need of extrapolation. For the used dataset it was needed to estimate the diameter at height zero. This estimated diameter influences the calculated volume and could therefore also influence the shown results and the drawn conclusions. Like the diameter at height zero, a hypothetical diameter e. g. 1 m below the ground could help some interpolation methods to find good shapes at the ends of the stem. If this diameter was used it needs to be documented how it was estimated.

The definition of where the stem begins needs to be considered. Prodan (1965) defines that the stem does not include the stump but gives no information how high the stump is. When a tree is felled the stump height might have positive relation with the tree size. This would have the consequence that parts of the stem volume of a young tree will not be included in the old tree. To avoid such problems e. g. Assmann (1961) defined that the stem begins at the ground. Maybe a refinement of this definition needs to be made when trees are standing on slopes.

Diameters at heights close together followed by regions with no measurement should be avoided as some interpolation methods propagate this short trend to long stem sections and could increase small measurement errors proportional to the section length of interpolation. If measurements exist which are closer together than x cm followed by a section with no measurement, the close together measurements could be aggregated to one data point to avoid this problem.

In regions, like the stem base, where the diameter is fast decreasing in a non-linear way shorter distances between the measurements would help to come to accurate volume calculations. The same is the case for small trees. A standard measurement of diameters every 2 m will lead on small trees to a number of less than 2–3 measurements what will lead to insecure volume calculations.

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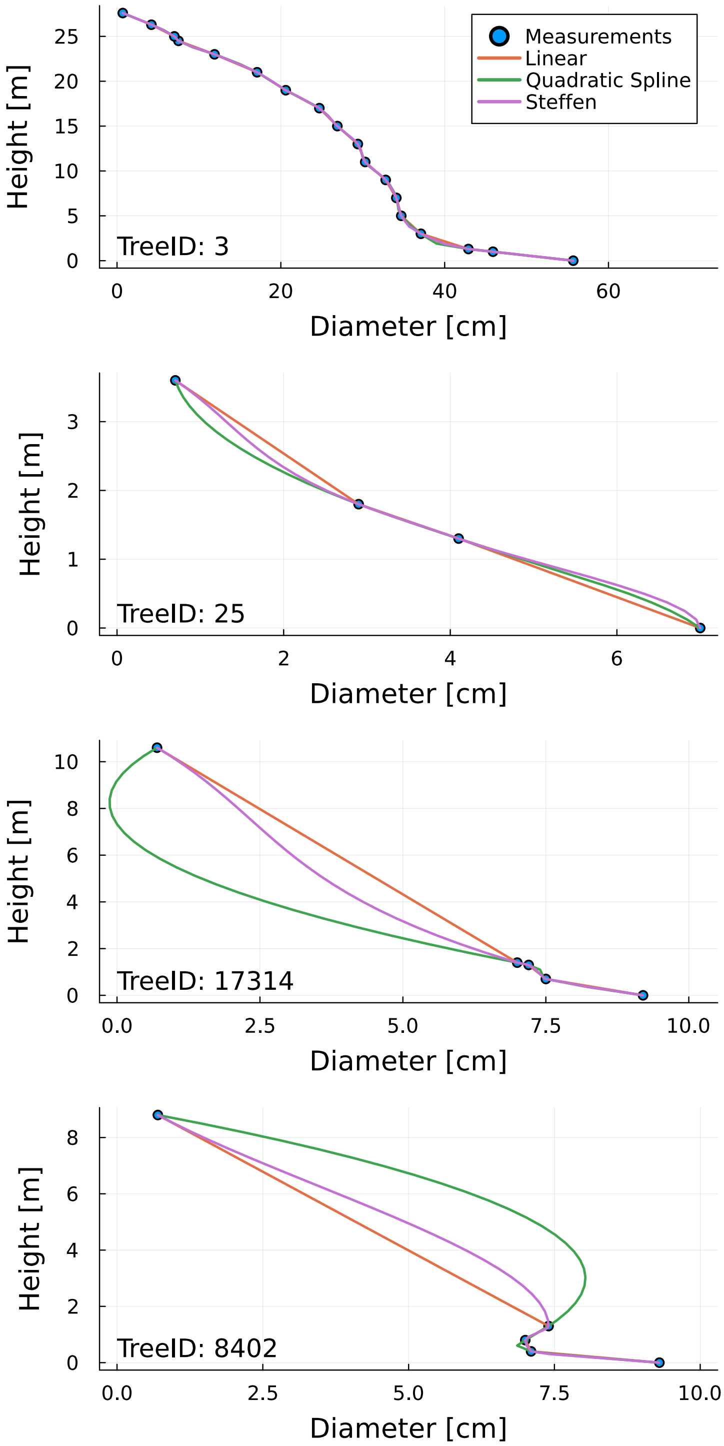

Figure 8: Examples of different interpolation between measured diameters.

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