

Comparison between Volume Measured by
Sections and Volume Estimated by Spiegel-
Relascop on Standing Trees

A. GOFAS

Preface

This work has been suggested by my Prof. Anton Georgopoulos, director of Forest management and Forest mensuration of the University of Thessaloniki (Department of Agriculture and Forest School).

He had accredited that a great help would be offered to the Forest Mensuration, if it would be possible to estimate the volume of a standing tree by Relascop with the required precision to the forest practice, so he charged me to investigate this problem. For this purpose I compared in reference to their precision the known methods of the volume estimation of a standing tree, using the Relascop with the method of the volume measurement by sections of a fallen tree, using caliper and tape, in the University Forest of Pertuli, during the summer of 1965.

I proceeded in this comparison because no one up to now has reffered the above in the existing bibliography, giving a solution to the problem in this way.

1. Introduction

The spiegel-relascop has been proved that it is an indispensable and multipurpose instrument for foresters, with which we can find out by indirect estimation the parameters of the volume of a standing tree or a stand, economically and with the required precision.

But with this instrument, it is obvious, that we optically estimate the characteristic of a tree or a stand, from a distance with an angle count or by the point and line sampling, so that a set of random and systematic errors are introduced which finally influence the degree of accuracy of the estimated parameters absolutely or per cent.

Consequently, we put up the question: With what precision can we estimate the characteristic of a tree or a stand by using the relascop?

Until now, as we know, the investigators (BITTERLICH, GROSENBAUGH, BLUTEL, HELLRIGL, GÜDE, HIRATA e.c.t.), have been worked more on the point sampling theory, by an angle count. They have also worked with the applicability of the instruments (Relascop, Prism, Conometer) that depends on this theory.

But, until now little is referred in the bibliography about the precision which we can achieve by using the above instruments for estimating a tree or a stand parameter.

HELLRIGL (1960, P. 14) relates, that UNTERDORFER estimating the number of the trees of a stand per ha by Relascop, has proved based on PRODAN's and HÄBERLE's statistical investigations that the total mean error is about $\pm 4\%$ and the maximum $\pm 11\%$.

The GIBBES and GARTER (1964, P. 580) have proved after using the relascop for height measurements taken on four species of Appalachian hardwoods that this relascop did not differ significantly from tape measurements on the same tree after felling. Average heights based on relascop measurements will probably be sufficiently accurate for general purposes.

Besides the above, up to nothing has been related in the existing bibliography about the successful accuracy regarding the estimation of the volume of a standing tree by sections or by the PRESSLER's formula by the relascop.

BITTERLICH (1958), BLUTEL (1962), HELLRIGL (1960), and PATRONE (1963) recommend the methods, by which we can find out the volume of a standing tree by sections or by PRESSLER's formula using BITTERLICH's Relascop, without referring to us the degree of accuracy we can succeed by using the above methods.

Consequently, the purpose of this research is to be shown: with which degree of accuracy we can estimate the volume of a standing tree by BITTERLICH's Relascop using the methods of

the volume measurements by sections and by PRESSLER's formula, compared with the method by sections using caliper and tape on the same tree after felling.

2. Material and Method of Research

For our study we chose the irregular selection forest of *Abies Hybridogenus* (*Abies Alba* X *Abies cephalonica*) of the University Forest of Pertuli.

The sample trees had been marked and they were going to fall down during the management year of 1965 by the individual selection system. From these trees that were about 850, a sample of 169 trees and 165 trees for each method, have been taken by systematic sampling and by the rules of statistical methods.

For each method every sample size, as we will see in the following, has been judged that it was very sufficient for a significant level 5 % and with a required precision of ± 5 %.

From this experiment material the volume of every one of the sample trees that have been taken as a sample, has been estimated by BITTERLICH's Relascop, using the methods by sections and the PRESSLER's formula and has been measured by caliper and tape after felling, as following (HELLRIGL 1960 p 15-18).

a) Cubic Volume by Sections (Method A)

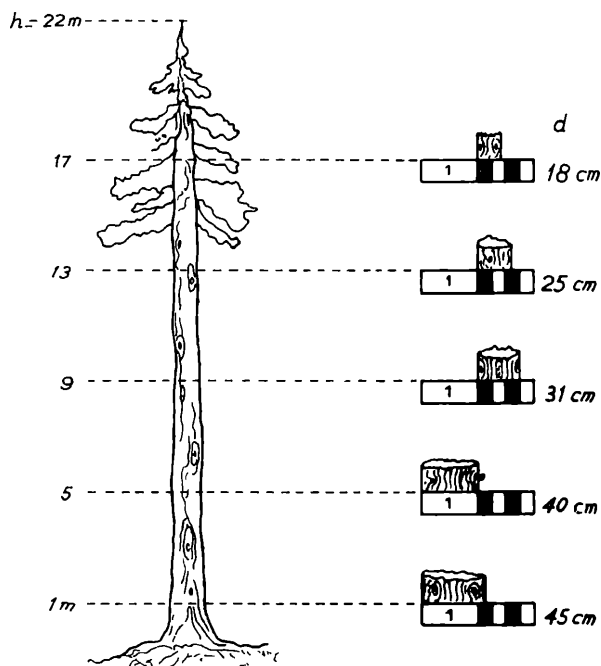
In this method we always use the basal area factor $F = 4$ of the instrument, after setting ourselves in a distance 15 m, 20 m, and 30 m, from the measured tree. So that from this distance we can estimate the diameter of a tree in different heights, if we have in mind that half of the basal area factor $F = 1$, gives an estimated diameter of a tree in cm, as the estimated distance in m between tree and point (figure 1).

On the base of this principle, the estimation of the diameter outside bark of each sample tree was done in heights 1 m, 5 m, 9 m, ... e.c.t. and height measure by the height scales 15 m, 20 m, 25 m, and 30 m of the instrument.

From these selected data the d.b.h. and the diameter in one meter height, outside bark have been taken as the average of the long and short axes.

Finally, if we consider a tree stem with diameters $d_1, d_5, d_9, \dots, d_n$ measured in m, at intervals of 1 meters along the stem, the volume of this stem by sections can be found by applying the formula as follows:

Fig. 1
Volume estimation by sections using Relascop



Measurement from a distance 20 m, by basal area factor $F=4$

Calculations

$$\frac{1}{3} \cdot 254 \times 500 = 42.333 \text{ cm}^3$$

$$\frac{254 + 491}{2} \times 400 = 149.000$$

$$\frac{491 + 755}{2} \times 400 = 249.200$$

$$\frac{755 + 1257}{2} \times 400 = 402.400$$

$$\frac{1257 + 1590}{2} \times 400 = 569.400$$

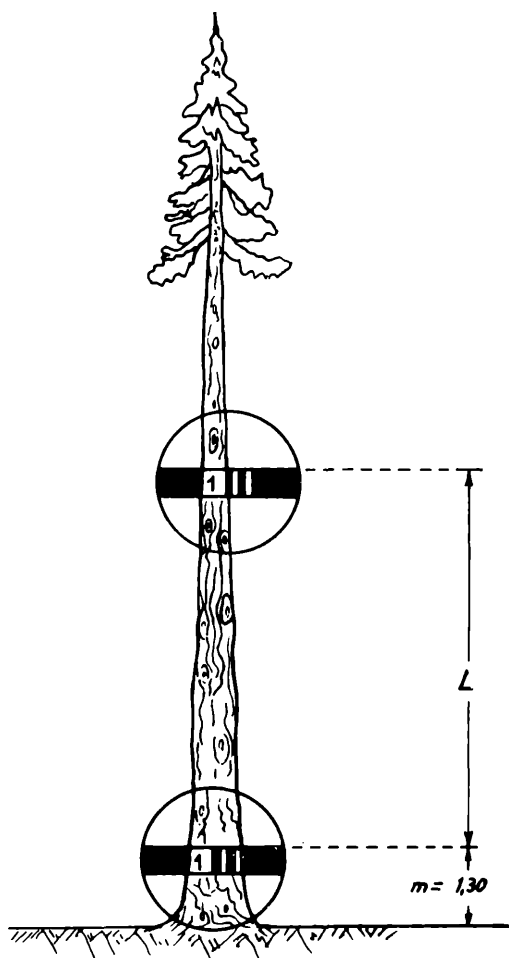
$$1590 \times 100 = 159.000$$

$$= 1571333 \text{ cm}^3$$

$$= 1,5713 \text{ m}^3$$

Fig. 2

Volume estimation by Relascop, using the Pressler's formula



Measurement of L on the height-scale 25 m

$$V = \frac{\pi}{4} l_1 \left(\frac{d_1^2}{2} + d_5^2 + \dots + \frac{d_n^2}{2} \right) + \frac{\pi}{4} l_1 \cdot d_1^2 + \frac{1}{3} \cdot \frac{\pi}{4} d_n^2 l_2$$

where l_1 4 m
 l_1 1 m and
 l_2 the length in meters of the top section.

b) Cubic Volume by PRESSLER's Formula (Method B)

For this purpose we select the basal area factor $F = 4$ of the instrument and we set ourselves in such a distance far from the tree, from which we must do the estimation of the volume.

From this distance the width of the above basal area factor must cover exactly the d.b.h of this tree (figure 2). After that we read the registered value on the height-scale 25 m of the instrument (fast reading).

Afterwards along the stem we seek again a point, the diameter of which covers exactly the width of the basal area factor $F = 4$ and on this point we read again the registered value on the same height-scale 25 m (second reading).

These two remarks are added or subtracted respectively, if these had been read in either side from the zero of the height-scale 25 m or to the same direction of it, giving a number L that is equal to:

$$L = \frac{2}{3} \cdot f \cdot h \quad (2)$$

With this number and with the measured d.b.h outside bark by the caliper, it was found out the volume of every tree by the formula:

$$V = \frac{\pi}{6} d^3 \cdot \left(L + \frac{3}{2} \cdot \frac{m}{d} \right) \quad (3)$$

where: m 1,3 m
 d the d.b.h.
 f the form factor and
 h the height of the tree.

c) Cubic Volume by Section Using Caliper and Tape (Method C)

After the end of the two describable measurements of the volume by the Relascop, every sample tree was fallen down. The volume of this tree was measured by sections outside bark using caliper and tape.

For this purpose the arithmetic average diameters of the $d_{\max}$

and the perpendicular to it d_p were taken, at intervals 1 4 m beginning from one meter level up to the ground.

With these means diameters $d_1, d_5, d_9, \dots$ measured in cm at intervals 1 m, 5 m, 9 m, e.t.c. up to the ground and from the total height, we have measured the real volume every sample tree by the formula 1 (GEORGOPOULOS p 14 - 16, 1963).^{x)}

3. Comparison Between Methods

In order to be shown, if there are significant differences between the above methods of the volume measurement, A - C and B - C compared by two, we calculate the differences of the sample trees volume that were taken as paired variates and after that we compare the means of these differences to zero (Null hypotheses, SNEDECOR p 49 and so on).

If there were no real differences in the volumes of the two groups A - C or B - C, it would be logical to say that the mean of each array of differences would be zero. On the other hand, the $\pm$ differences for each pair would be normally distributed about a mean of zero.

But the table I show below that the sum of these differences, on 169 and 165 pairs of measurements are respectively not zero. Consequently, it must be shown further, if this total difference of each compared method is or not significant, at a chosen significant level.

For this purpose we must calculate:

- a) The mean of the differences

$$\bar{X}_d = \bar{X}_1 - \bar{X}_2 \quad (4)$$

- b) The standard deviation of the differences

$$S^2 = \frac{\sum (X_1 - X_2)^2 - \left\{ \left[\sum (X_1 - X_2) \right]^2 / n \right\}}{n - 1} \quad (5)$$

- c) The standard error of the differences

$$S_d = \frac{S}{\sqrt{n}} \quad (6)$$

x) The analytical elements of this work are deposited to the Laboratory of Forest Management and Forest Mensuration.

With the above statistical numbers we proceed further to the comparing of the mean volume of the differences, using the t-test, by the formula

$$t = \frac{\bar{X}_1 - \bar{X}_2}{S_d}$$

The calculated value of t compared with the theoretical value of $t_{0,05}$, for degrees of freedom $n - 1$ and probability 95 %, will show us if there are significant differences between the compared methods of the volume measurement (HUSCH 1963 p 375 - 376).

a) Comparison Between A and C Method

For this purpose we calculate the differences of the trees volume by two, which have been taken as a sample and from these pairs we compare the mean of the differences to zero.

The table 1 shows us that the estimated total volume by sections (method A), using the Relascop is $\Sigma V_1 = 305,3713 \text{ m}^3$ and the measured one by sections (method C), using caliper and tape is $\Sigma V_3 = 300,3285 \text{ m}^3$, on 169 pairs of observations. Similarly the sum and the square of the total difference between the compared methods A - C are $\Sigma (V_1 - V_3) = 5,0428 \text{ m}^3$ and $\Sigma (V_1 - V_3)^2 = 12,2098 \text{ m}^3$ respectively. This total difference must be shown, if it is or not significant, at a chosen significant level 0,05.

The results of the statistical calculations, between the two compared methods A - C show us that these are the following (Tabel II):

The mean of the differences $\bar{V}_d = 0,02983 \text{ m}^3$

The standard deviation of the differences

$S_V = \pm 0,2670 \text{ m}^3$ and

The standard error of the differences $S_{\bar{V}} = 0,02054 \text{ m}^3$.

With the above statistical numbers, it has been proved by the formula (7) that the t criterion is $t = 1,452$. This calculated value of t = 1,452 compared with the theoretical value of $t_{0,05}$ 1,96 that can be found out on a statistical table (SNEDECOR 1961, p 46), for degrees of freedom $n - 1 = 168$ and probability 95 % shows that $t < t_{0,05}$. From this statistical result we are led to the conclusion that there are not significant differences between the two compared methods and the null hypothesis is not rejected. Consequently the method of the volume measurement by caliper and tape of a tree is possible to be substituted by the other that estimates the volume of the tree by sections with the Relascop.

Sample size	The sum and the square of the differences between the methods		
	Method A The estimated total volume by sections using Relascop	Method B The estimated total volume by Pressler's formula using Relascop	Method C The measured total volume by sections using caliper and tape
	ΣV_1	ΣV_2	ΣV_3
169	305,3713 m ³		300,3285 m ³
			+ 5,0428 m ³
			12,2098 m ³
165		260,6997 m ³	294,6117 m ³
			- 33,9120 m ³
			34,7548 m ³

Compared Methods	The sample size	The mean of the differences	The standard deviation of the differences	The standard error of the differences	
A -	169	0,02983 m ³	+ 0,2670 m ³	0,02054 m ³	1,452
B -	165	0,20553 m ³	+ 0,4110 m ³	0,03199 m ³	6,424
					1,96

Compared methods	For a significant level 5 %		The coefficient of variation	The sample size for a worthwhile difference 5 % of the mean real volume. Significant level 5 %
	The calculated population difference	The required population difference		
A - C	0,04023 m ³	0,0888 m ³	15 %	36
B - C	0,06894 m ³	0,0840 m ³	27 %	112

Thus during the estimation of a volume by section of a standing tree, with the Relascop, we overestimate it, in opposition to the estimated volume that we measure by caliper and tape on the same tree after felling. The size of this positive error, partly is a random one (error of reading, error of observation e.c.t.), which is neutralized by the increment of the measurements, and partly is a systematic one, that cannot be zero with the increment of the measurements.

But this systematic error mainly is due to the following causes: As it has been observed in the action, the projected angle count by the Relascop on a volume measured tree from a distance, in order to estimate the diameters by sections in different heights from the ground, usually overestimates these diameters of the upper-heights.

This is evidently due to the fact that the critical angle is always touched on the tree in acute angle to this axis and lower than the real height of calipering. Consequently every time the diameter is estimated a little over the real one.

The subjective selection of the sight of the tree from which we estimate the volume of a standing tree always gives overestimated upper heights diameters, because of their increment to the points contribution of branches, where they are touched by the angle count sides.

If we examine every difference of the observations, we accredit that trees with many branches and with infections by mistletoes, show differences of volume which are between + 15 % and + 25 % than the real one.

The sample size that has been taken 169 trees, is greater than the required one, for a significant level 5 % and accuracy ± 5 % of the mean real volume. This can easily be proved by the formula:

$$\delta \frac{S_v \cdot t_{0,05}}{\sqrt{n}} \quad (8)$$

Where: δ The population difference from the mean real volume
 S_v The standard deviation of the differences and
 n The sample size (SNEDECOR 1961, p 60, section 2.15).

From the above formula (8) it has been proved (table III) that for a significant level 5 % the calculated population difference

is, $\delta = 0,04023 \text{ m}^3$ and the required one, $\delta_1 = t_{0,05} \cdot s_{\bar{v}} = 0,05 \cdot 1,777 = 0,0888 \text{ m}^3$ respectively. Thus the sample size with $\delta_1 = 0,0888 \text{ m}^3$ and for a required precision $\pm 5 \%$ of the mean real volume, would be 36 trees and not 169 trees.

b) Comparison between B and C method

We also calculate the differences of the trees volume by two, which have been taken as a sample and from these pairs we compare the mean of the differences to zero.

From the table I is shown again that the estimated total volume by PRESSLER's formula (Method B), using the Relascop is

$\Sigma V_2 = 260,6997 \text{ m}^3$ and the measured one by sections (method C), using caliper and tape is, $\Sigma V_3 = 294,6117 \text{ m}^3$, on 165 pairs of observations. Similarly the sum and the square of the total difference between the compared methods B - C are

$$\Sigma(V_2 - V_3) = -33,9120 \text{ m}^3 \text{ and } \Sigma(V_2 - V_3)^2 = 34,7548 \text{ m}^6$$

respectively.

Consequently it will have to be shown again if this total difference is or not significant at a chosen significant level 5 %.

In order to prove the above we calculate the following statistical numbers (table II):

The mean of the differences	$\bar{V}_d$	$0,20553 \text{ m}^3$
The standard deviation of the differences	s_v	$0,4110 \text{ m}^3$
The standard error of the differences	$s_{\bar{v}}$	$0,03199 \text{ m}^3$

With the above calculated elements it has been proved by the formula (7) that the t criterion is $t = 6,424$. This calculated value compared with the theoretical value of $t_{0,05} = 1,96$ which can be found out, on a statistical table for degrees of freedom $n - 1 = 164$ and with a probability 95 %, shows that $t > t_{0,05}$. Consequently we are led to the conclusion that there are significant differences between the two compared methods B - C and the null hypotheses is rejected. Thus we cannot use the Relascop to estimate the volume of a standing tree, when with this instrument we get the volume by the PRESSLER's formula.

From the above formula (8) it has been proved (table III) that for a significant level 5 % the calculated population difference is again $\delta = 0,06894 \text{ m}^3$ and the required one $\delta_1 = 0,0840 \text{ m}^3$ respectively. Thus the sample size with $\delta_1 = 0,0840 \text{ m}^3$ and for a required precision $\pm 5 \%$ of the mean real volume, would

be 112 trees and not 165 trees.

4. General Conclusions

As we have proved, by BITTERLICH's Relascop, we can estimate the real volume of standing trees with the required precision to the forest practice and research using the method by sections and not the method by PRESSLER's formula.

Using the above method to estimate the volume of the trees, we have the following advantages:

- a) We estimate the volume of standing trees, not after felling them. Thus we don't destroy the forest and we don't disturb its management.
- b) The forest owner using this instrument for this purpose, saves up time and money without sacrificing the required precision.

Vergleiche zwischen der sektionsweisen Kubierung von Stämmen und deren Schätzung mit dem Spiegelrelaskop von Bitterlich

Prof. Dr. A. GEORGOPOULOS, der Leiter des Institutes für Forsteinrichtung und Dendrometrie der Aristoteleion-Universität, hat uns beauftragt, die Genauigkeit der Schätzung des Volumens der stehenden Bäume durch das Spiegelrelaskop von BITTERLICH, mit Hilfe sowohl der Durchmessermessung in verschiedenen Höhen (b-Messung, Methode A), wie auch nach der unmittelbaren Formhöhenmessung (fh/d-Messung, Methode B), zu untersuchen. Zu diesem Zweck haben wir im Juni und September 1965 im Lehrwald von Pertuli^{x)} von 850 angezeichneten Bäumen in der Abteilung "Wathy" 169 bzw. 165 Stämme systematisch ausgewählt und deren Volumen vor der Fällung nach der b-Messung bzw. fh/d-Messung berechnet. Nach deren Fällung haben wir den Inhalt dieser Stämme durch sektionsweise Kubierung genau ermittelt (Methode C) und das Ergebnis mit den Methoden A und B verglichen (siehe Tab. I)^{xx)}.

Der Vergleich zwischen den Durchschnitten der Methoden A und C einerseits und B und C andererseits mittels der t-Verteilung nach STUDENT hat ergeben, daß die b-Messung sich von liegenden (genaueren) Stamminhaltsmessung nur zufällig unterscheidet, während die fh/d-Messung und die Messung an liegenden Stämmen wesentlich voneinander abweichen. Die statistische Sicherheit ergibt 95 %, d.h. Unterschreitungswahrscheinlichkeit 5 % (siehe Tab. II). Zum Schluß haben wir für eine Genauigkeit des Mittelwertes ± 5 % innerhalb einer statistischen Sicherheit von 95 % die nötige Mindestzahl an Messungen für beide Methoden A und B berechnet (siehe Tab. III).

Aus diesen Untersuchungen geht hervor, daß das Spiegelrelaskop von BITTERLICH ein wertvolles und rasch arbeitendes Forstinstrument für die genaue Schätzung des Stamminhaltes stehender Bäume darstellt, unter der Voraussetzung, daß eine Anzahl von Stämmen gemessen werden müssen, um den Fehler des Mittelwertes in bestimmten Grenzen zu halten. Infolgedessen ist es auch möglich, Massentarife in einem Wald aufzustellen, ohne die Bäume der Stichprobe fällen zu müssen.

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- x) Dies ist ein plenterartiger Tannenwald, der den Übungen der Forststudenten dient.
- xx) Originalwerte sind im Institut für Forsteinrichtung und Dendrometrie der Aristoteleion Universität hinterlegt.

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Anschrift des Verfassers

Dr. A. GOFAS
Aristoteleion Universität
Thessaloniki
Griechenland