FOREST TRANSITION AS A STOCHASTIC PROCESS

Tasiti SUZUKI

When we wish to describe the state of forest stands mathematically , we used to represent it by a histogram of their diameter distribution Then if we want to research diameter growth, we must consider laws of motion of the histogram. The aim of this paper is to derive a basic equation of these laws. It is not necessary to use many words for the importance of this problem in the forest research.

1. Before we consider forest stands which consist of many trees , we think of an individual tree which is an element of the stands. Now we suppose a tree whose diameter is x cm at time t With an elapse of time , at time τ its diameter will be bigger than x cm in general. Let it be y cm. But tree growth is governed by some randomness which are unknown for us. So we cannot specify y cm determinately, but we can only predict it in probability. Hence we shall define a transition probability $p(t,x;\tau,y)$ that a tree of x cm diameter at time τ will put on y cm at time τ

If we assume this transition to be Markov process, an equation to determine the probability $p(t,x;\tau,y)$ is , in general, a Kolmogorov equation. But we must think of two different transitions, one is the continuous corresponding to diameter increase and another is the discontinuous corresponding to tree death.

In convenience we shall treat this tree death as a jumpping to 0 *cm* from any x *cm*. Then we shall assume that As a function of y $p(\tau,y;\tau+\Delta\tau,z)$ is continuous in the neighborhood of y, but it is singular at $z=0$ and is the exact likeness of Dirac's δ-function there.

Under these assumptions we can put

$$\lim_{\Delta\tau\to 0}\frac{1}{\Delta\tau}\int_{|y-z|>\varepsilon} p(\tau,y;\tau+\Delta\tau,z)dz = p(\tau,y;\tau+\Delta\tau,0) = \gamma(\tau,y) \tag{1}$$

And we assume also for any function $R(z)$ that

$$\lim_{\Delta\tau\to 0}\frac{1}{\Delta\tau}\int_{|y-z|>\varepsilon} p(\tau,y;\tau+\Delta\tau,z)R(z)dz = \gamma(\tau,y)R(0) \tag{2}$$

In these cases $\gamma(\tau,y)$ means the probability that a tree of y *cm* diameter is dead for an unit time interval from $\tau^{\sim}$.

Then we define two functions,

$$\alpha(\tau,y) = \lim_{\Delta\tau\to 0}\frac{1}{\Delta\tau}\int_{|y-z|<\varepsilon} (z-y)^2 p(\tau,y;\tau+\Delta\tau,z)dz \tag{3}$$

$$\beta(\tau,y) = \lim_{\Delta\tau\to 0}\frac{1}{\Delta\tau}\int_{|y-z|<\varepsilon} (z-y) p(\tau,y;\tau+\Delta\tau,z)dz \tag{4}$$

From eq. (1) we have

$$\int_{|y-z|<\varepsilon} p(t,y;\tau+\Delta\tau,z)dz = 1 - \gamma(\tau,y) \to 1$$

Further we define a conditioned transition probability from x to y excepting dead trees.

$$q(\tau,x;\tau+\Delta\tau,y) = p(\tau,x;\tau+\Delta\tau,y)\Big/\int_{|y-x|<\varepsilon} p(\tau,x;\tau+\Delta\tau,y)$$

$$\text{when } |y-x|<\varepsilon \,;$$

$$q(\tau,x;\tau+\Delta\tau,y) = 0\,, \qquad \text{when } |y-x|>\varepsilon$$

Now we have

$$\int_{|y-x|<\varepsilon} q(\tau,x;\tau+\Delta\tau,y)dy = \int_0^{\infty} q(\tau,x;\tau+\Delta\tau,y)dy = 1\,,$$

$$\int_{|y-x|<\varepsilon} f(x,y)q(\tau,x;\tau+\Delta\tau,y)dy = \int_0^{\infty} f(x,y)q(\tau,x;\tau+\Delta\tau,y)$$

Using these relations we can show that $\alpha(\tau,y)$ is equal to the rate of changes of diameter variances and $\beta(\tau,y)$ is equal to the increase rate of average diameter, namely ,

$$\alpha(\tau,y) = \lim_{\Delta\tau\to o} \frac{1}{\Delta\tau} \{\int_o^\infty (y-\bar{y})^2\phi(\tau+\Delta\tau,y)dy - \int_o^\infty (x-\bar{x})^2\phi(\tau,x)dx\} \tag{5}$$

$$\beta(\tau,y) = \lim_{\Delta\tau\to o} \frac{1}{\Delta\tau} \{\int_o^\infty y\phi(\tau+\Delta\tau,y)dy - \int_o^\infty x\phi(\tau,x)dx\} \tag{6}$$

If these coefficients are all given, the forest transition pro -bability satisfies the following equation.

$$\frac{\partial}{\partial\tau} p(t,x;\tau,y) = \frac{1}{2}\frac{\partial^2}{\partial y^2}[\alpha(\tau,y)p(t,x;\tau,y)] - \frac{\partial}{\partial y}[\beta(\tau,y)p(t,x;\tau,y)] - \gamma(\tau,y)p(t,x;\tau,y) \tag{7}$$

This is the so-called Kolmogorov equation except that it contains the third term corresponding to tree death. We can refer to his famous article ;*Über die analytische Methoden in der Wahrscheinlich -keitsrechnung. Math. Ann. 1931.*

2. Functional space of diameter distribution.

In the study of function $f(x)$ of a variable x, it is sometimes convenient to regard it as a infinite dimensional vector

We consider a histogram with n ranks as n-dimensional vector

Similiarly , we can consider a usual function as a infinite dimensional vector which is the limitting histogram as n tend to infinite

Such a way of looking is rather intuitive than logically exact

But from this point of view we can use the geometrical analogy to the n-dimensional vector space

We call this infinite dimensional space a functional space

We shall describe a distribution of forest diameter x at any time t by a function $\phi(t,x)$, and then consider a functional space Φ whose elements are these functions $\phi(t,x)$

Now we can take a view that $\phi(t,x)$ is a point in Φ and it flows in Φ with the lapse of time Such a flow defines a stochastic process In this case our way of thinking is as same as that of " Forest age space

Cf. T. Suzuki ; *An application of Markov chains in forestry , Gentan probability* , IUFRO Section 25 , Meeting at Birmensdorf. September 16-18, 1970

Suppose the diameter distribution at time $t+\tau$ to be $\phi(t+\tau,y)$

The above stated flow $\phi(t,x) \rightarrow \phi(t+\tau,y)$ defines a representation in Φ Writing this representation S_τ , we can describe this flow as follows ;

$$S_\tau(\phi(t,x)) = \phi(t+\tau,y) \in \Phi$$

Using such a representation we can write a distribution after time $\tau_1+\tau_2$,

$$S_{\tau_1+\tau_2}(\phi(t,x)) \in \Phi$$

Also we may write this

$$S_{\tau_2}\{S_{\tau_1}(\phi(t,x))\} \in \Phi$$

Then we have a law of composition ,

$$S_{\tau_1+\tau_2} = S_{\tau_2} \cdot S_{\tau_1} \qquad (8)$$

and it is also commutative ,

$$S_{\tau_1} \cdot S_{\tau_2} = S_{\tau_2} \cdot S_{\tau_1}$$

From the definition of S_τ it is obvious ,

$$S_o(\phi(t,x)) = \phi(t,x)$$

namely , S_o is an identity representation

But there must not exist the inverse $S_{-\tau}$ for any representation S_τ , for the nature must be not reversible in itself

Therefore , a set of representation S_τ constitutes a semi-group according to the above defined composition rule

Now consider S_τ in the concrete For an occurrence that a tree of *x cm* diameter at time *t* becomes *ycm* across at time τ, we have defined a transition probability $p(t,x;\tau,y)$

We easily have from the very definition of the probability ,

$$\phi(\tau,y) = \int_0^\infty \phi(t,x)p(t,x;\tau,y)dx \qquad (9)$$

On the other hand we can write as

$$\phi(\tau,y) = S_{\tau-t}\{\phi(t,x)\}$$

Thereby we have in mind the concrete meaning $S_{\tau-t}$ as an opera -tion $\int_0^\infty * \, p(t,x;\tau,y)dx$

Forest stands are convolution of such an individual tree If we differentiate eq. (9) and then use eq. (7)

$$\frac{\partial}{\partial\tau}\phi(\tau,y) = \frac{1}{2}\frac{\partial^2}{\partial y^2}\{\alpha(\tau,y)\phi(\tau,y)\} - \frac{\partial}{\partial y}\{\beta(\tau,y)\phi(\tau,y)\} - \gamma(\tau,y)\phi(\tau,y) \qquad (10)$$

This is the basic equation which gives the laws of motion of $\phi(\tau,y)$.

3. Assumption to the coefficient functions

In the research of the tree growth in terms of a definite differ -ence diagram , we found that all points for average diameter form neatly in a line , namely they obey the relation

$$x(t+1) = px(t) + q$$

where $x(t+1)$, $x(t)$ are the average diameter at time $t+1$ or t , respectively

From this relation we can prove that a growth curve of the average diameter obeys the so-called Mitscherlich's curve This fact shows that the average diameter growth of a forest is an occurrence like that of one molecular chemical reaction

For an individual tree we must account of random deflections so we think of

$$dx / dt = k(M-x) + f(t) \tag{11}$$

where $f(t)$ means a random deflection at time t and average of $f(t)$ is assumed to be 0 Solving this equation and averaging the result , we have

$$\overline{x(t)} = M [1 - L \exp(-kt)] \tag{12}$$

namely this is a Mitscherlich's curve

Then assuming that

$$\overline{f(\tau)^2} = \overline{f(\tau+\xi)^2} = \overline{f^2}$$

we have for the correlation coefficient $r(\xi)$

$$r(\xi) = \overline{f(\tau)f(\tau+\xi)}/\overline{f^2}$$

Now thinking of that the correlation between $f(\tau)$ and $f(\tau+\xi)$ decreases rapidly to 0 as time ξ goes on , we have an expression for the diameter deviation

$$\sigma^2 = \{1 - \exp(-2kt)\}\int\overline{f^2}\cdot r(\xi)d\xi/k \tag{13}$$

Differentiating eq. (12) and eq. (13) , we have two expressions

$$\alpha(\tau,y) = \text{const } (M - \overline{y})^2 = 4a_1^2ke^{-2k\tau} \tag{14}$$

$$\beta(\tau,y) = \text{const } (M - \overline{y}) = b_1ke^{-k\tau} \tag{15}$$

And for brevity we put the probability of tree death constant

$$\gamma(\tau,y) = c = \text{const} \tag{16}$$

This means that a number of the dying trees will be proportional to the total number of living trees and to the length of the time interval under consideration

In this case taking a new function ψ such that

$$\phi(\tau,y) = e^{-c\tau}\psi(\tau,y)$$

in which $\exp(-c\tau)$ means the decrease of a total amount of trees , a new equation for $\psi(\tau,y)$ will be

$$\frac{\partial}{\partial\tau}\psi(\tau,y) = \frac{1}{2}\frac{\partial^2}{\partial y^2}\{\alpha(\tau,y)\psi(\tau,y)\} - \frac{\partial}{\partial y}\{\beta(\tau,y)\psi(\tau,y)\} \qquad (17)$$

4. Solution of the equation

Substituting these coefficient functions of eq. (14) (15) and (16) the equation (17) becomes the so-called Bachelier equation

$$\frac{\partial\psi}{\partial\tau} = 2a_1^2ke^{-2k\tau}\frac{\partial^2\psi}{\partial y^2} - b_1ke^{-k\tau}\frac{\partial\psi}{\partial y} \qquad (18)$$

Through the following transformation of independent variables devised by Kolmogorov

$$t = \int_0^\tau 2a_1^2ke^{-2k\tau}du = a_1^2(1 - e^{-2k\tau})$$

$$x = y - \int_0^\tau b_1ke^{-ku}du = y - b_1(1 - e^{-k\tau})$$

Eq. (18) is transformed into the simplest and well-known heat induction equation

$$\frac{\partial\psi}{\partial t} = \frac{\partial^2\psi}{\partial x^2}\,. \qquad (19)$$

Among solutions of equation (19) the basic one which has δ-function as its initial distribution for $t = 0$ is

$$\psi_0(t,x) = \frac{1}{2\sqrt{\pi t}}e^{-\frac{x^2}{4t}}$$

Reversing the variables , we get the basic solution for eq. (18).

$$\psi_0(\tau,y) = \frac{1}{2\sqrt{a_1^2\pi(1-e^{-2k\tau})}}\exp\left(-\frac{\{y-b_1(1-e^{-k\tau})\}^2}{4a_1^2(1-e^{-2k\tau})}\right). \qquad (20)$$

, which shows that distribution of diameter y is normal and that its average and variance are determined by eq. (12) and eq. (13) respectively

Moreover , solution satisfying any initial condition

$$\psi(0,y) = f(y) \tag{21}$$

can be obtained by means of convoluting it to the basic solution Multiplying it by exp(-cτ) meaning decrease in tree number , lastly we are also able to have a solution of eq. (10) in integral form

$$\phi(\tau,y) = \frac{e^{-c\tau}}{2\sqrt{a_1^2\pi(1-e^{-2k\tau})}} \int_{-\infty}^{\infty} \exp\left(-\frac{\{y-b_1(1-e^{-k\tau})-\xi\}^2}{4a_1^2(1-e^{-2k\tau})}\right) f(\xi)d\xi \tag{22}$$

5. Recent development of the theory

(a) On the above stated we have concluded that a forest stand theoretically will grow obeying to normal distribution But there is another interesting case where we suppose the coefficient func -tions

$$\alpha(\tau,y) = \text{const } (M - y)$$

$$\beta(\tau,y) = \text{const } (M - y)$$

different from eq. (14) and eq. (15) On this case the solution is

$$\phi(\tau,y) = \frac{e^{(2a^2+b-c)\tau}}{\sqrt{2a^2\tau}\ M} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left\{\frac{(3a^2+b)\tau+\log(1-y/M)}{\sqrt{2a^2\tau}}\right\}^2\right)$$

which is gained by means of *Laplace transformations* In this case the forest will become uniform in the course of years and lastly will be a δ-function like initial distribution

(b). Out of these arguments , k and b are determined by the finite difference diagram On the diagram exp($-k$) is equal to a tangent of the finite difference line and this line intersects the bisector of the both axis at M Further , using this k we gain a from eq. (14) Moreover , the argument c which is the rate of decrease of total tree number is estimated by a tangent of declining line on the semi-logarithmic section paper   Our seminar are collecting these data of various types of the forests

(c). In order that we may apply the results for research of forests , we must solve the equation numerically   Mr. Umemura in our semi-nary room treated the equations by mesh method and succeeded to gain results well adopted for actual data

(d). To apply mesh method we suppose both time and diameter to be discrete and use finite differences as substitutes for derivatives. Assuming diameter to be discrete , we can consider diameter growth as a transition between diameter ranks   Now we think of a vector whose components are tree numbers belonging to each diameter rank   Namely   this vector is nothing but a diameter histogram of the forest   Then we gain a Markov chains between those diameter vect-ors which are operated by matrices whose components are probabili-ties between diameter ranks   These constructions are the same as that of the theory of the   Forest age space   In this case it is reduced to a problem to determine the diameter transition probabili-ty at any time τ The (j,k) element of the matrix denoted by $p(j,k)$ means the probability of transition from j *cm* rank to k *cm* rank   From the very definition we have

$$\sum_{k \geq j} p(j,k) = 1 ,$$

$$\sum_{k \geq j} (k-j)^2 p(j,k) = \alpha(\tau,y) - \beta(\tau,y)^2$$

$$\sum_{k \geq j} (k-j) p(j,k) = \beta(\tau,y)$$

These three relations are not sufficient to determine all the components of the j rank   But for small scale of time interval it is assumed that almost all elements of p(j,k) may be zero except several elements $k = j$, j+1, j+2, Supplement of some infor-mation afford to determine all components of the j th rank We have no conclusion for the best determination of the matrix

(e). The above stated method is to predict an average estimation in terms of multiplying tree numbers by probabilities If we simu-late the actual transition corresponding to each probability , we will gain an individual model which is not average and diverges for each experiment Mr. Sueta in our postgraduate course executed these simulation tests for a simple model forest It is found that the simulation method is satisfactory to solve the diameter transi-tion problem The following is an example of our Owase experimen-tal forest *Chamaecyparis obtusa* planted in 1908 Out of the data for 11 years from 1933 to 1943 we determined parameters relating to diameter increment Then we estimated the forest in 1951 from the initial stands in 1943 by means of the matrix method

The matrix components for one year are

$$p(j,j\ \ ) = 0.385 \sim 0.390$$

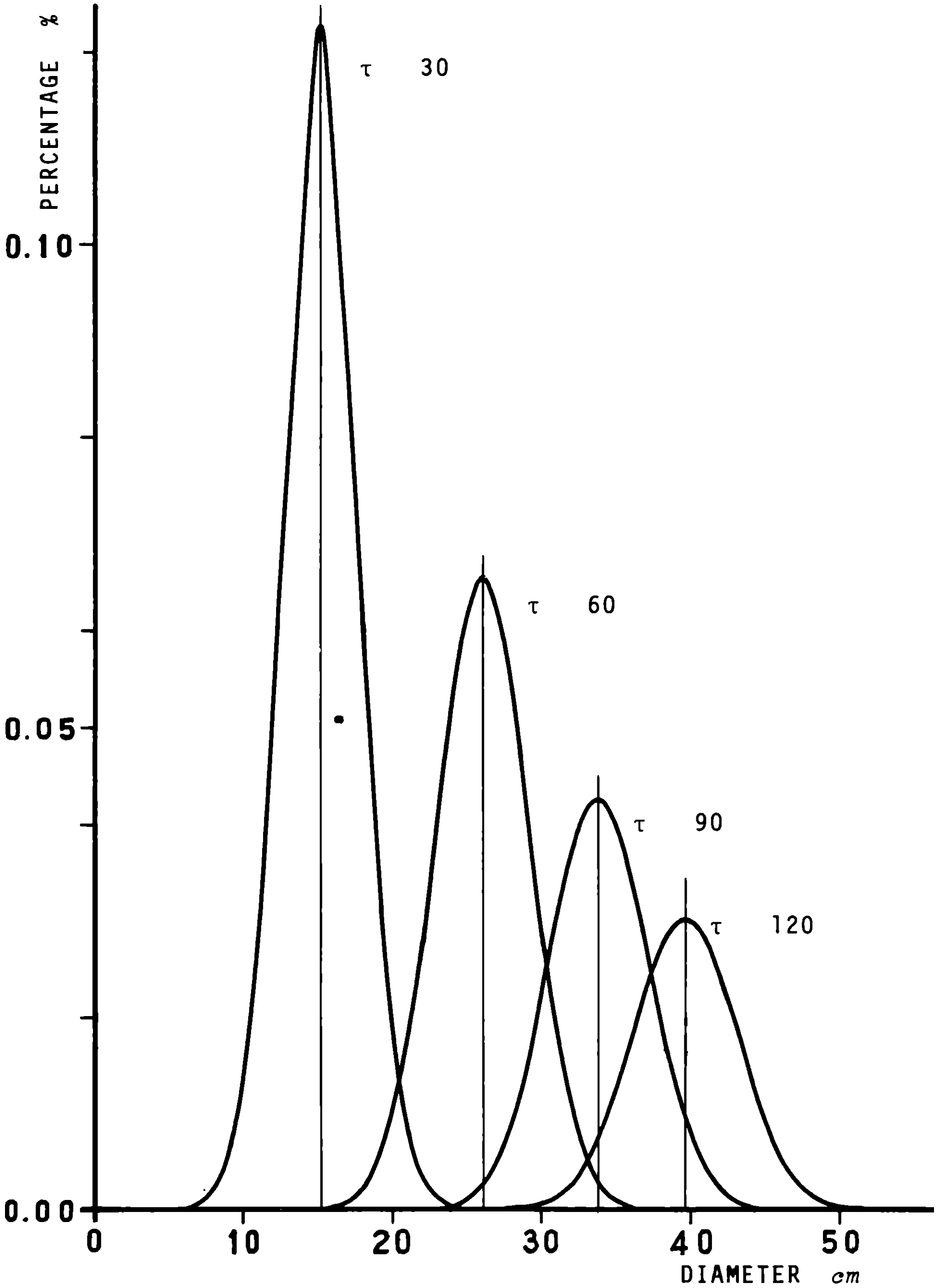
$$p(j,j+1) = 0.422 \sim 0.473$$

$$p(j,j+2) = 0.193 \sim 0.137$$

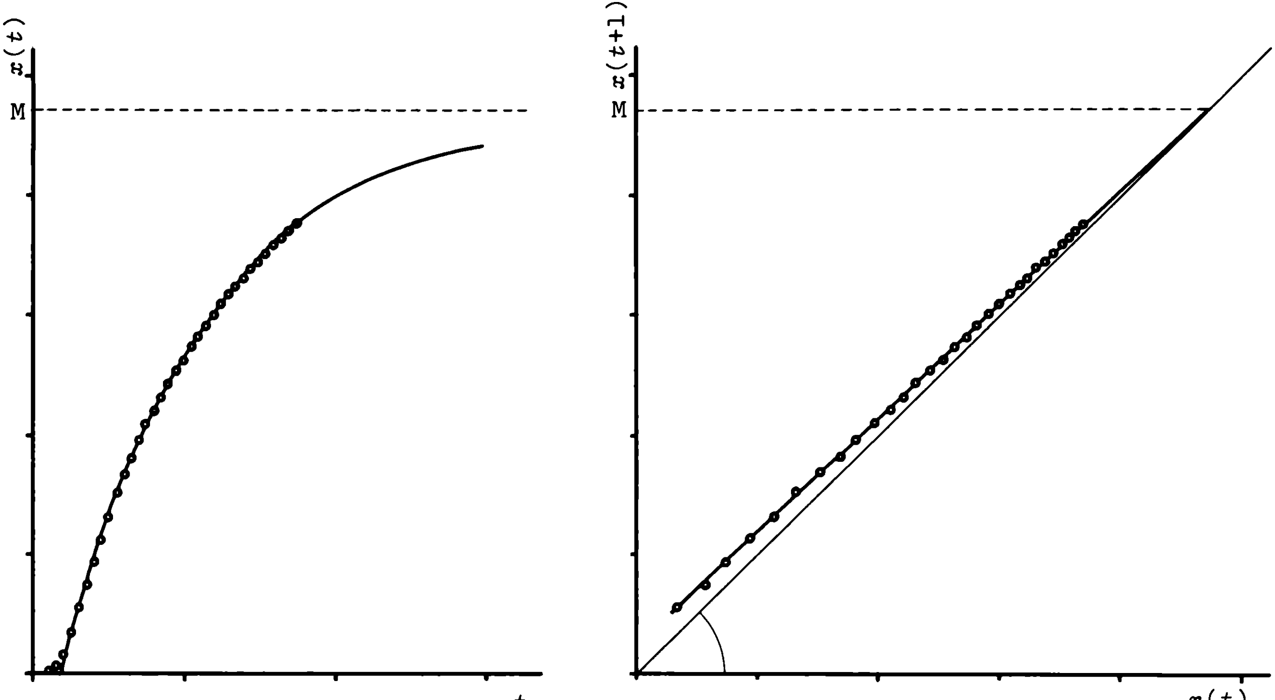
where diameter ranks are of the length 5 *mm* We can verify that the matrix estimate is well-fitted And the Monte Carlo experi-ments show that the difference between the actual stands and the estimated can be thought of not essential

MONTE CARLO EXPERIMENTS OF THE FOREST TRANSITION

diameter rank	initial forest in '43	the estimated in '51	Monte Carlo experiments of the forest transition exp.1	exp.2	exp.3	actual forest in '51
11 cm	1					
12	6		1			
13	18		1		1	
14	46	2	2		2	
15	68	6	7	6	10	4
16	77	16	19	10	14	16
17	73	31	29	20	31	30
18	74	45	43	44	37	47
19	48	52	44	59	56	48
20	36	54	60	49	53	61
21	34	49	53	44	46	44
22	13	40	35	43	38	39
23	9	30	28	41	33	32
24	4	22	25	27	23	18
25	3	14	14	12	14	16
26	2	8	7	9	12	9
27	2	4	5	7	4	5
28	1	3	3	5	2	5
29		2	1	2	3	2
30		1	2	1		3
31		1	1			1
32					1	
33				1		
average	17.34	20.46	20.43	20.86	20.50	20.64
variance	7.440	8.285	8.834	8.360	8.787	8.831



An example of forest transition from the δ-function like initial forest ;
a_1^2=6.7 , b_1=54 , k=0.011 , c=0.011



Growth curve *Difference diagram*

M=23.64, *k*=0.050, L=1.2

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Address for correspondence:

Prof. Dr. Tasiti SUZUKI
Faculty of Agriculture, Department of Forestry
Nagoya University

NAGOYA, AITI
Japan

FOREST TRANSITION AS A STOCHASTIC PROZESS

Tasiti Suzuki

Summary

We can express the state of forest stands by means of their diameter distribution $\phi(\tau,y)$ at time τ. The aim of this paper is to derive an equation of $\phi(\tau,y)$. Under some considerations we obtained a kind of Kolmogorov equation

$$\frac{\partial\phi(\tau,y)}{\partial\tau} = \frac{1}{2}\frac{\partial^2}{\partial y^2}[\alpha(\tau,y)\phi(\tau,y)] - \frac{\partial}{\partial y}[\beta(\tau,y)\phi(\tau,y)] - \gamma(\tau,y)\phi(\tau,y) ,$$

where $\alpha(\tau,y)$ is the rate of changes of diameter variances, $\beta(\tau,y)$ is the increase rate of average diameter and $\gamma(\tau,y)$ is the probability that a tree of y cm diameter will die for a unit time interval from τ.

From the natural observations of the forests we can induce that

$$\alpha(\tau,y) = const\ (M-\bar{y})^2,$$

$$\beta(\tau,y) = const\ (M-\bar{y}),$$

$$\gamma(\tau,y) = const$$

For these coefficients the above equation becomes the so-called Bachelier equation. It has a solution

$$\phi_0(\tau,y) = \frac{e^{-c\tau}}{2\sqrt{a_1^2\pi(1-e^{-2k\tau})}}\,exp\left(-\frac{\{y-b_1(1-e^{-k\tau})\}^2}{4a_1^2(1-e^{-2k\tau})}\right) ,$$

if the initial state has a δ-function like distribution.

Simulation by means of Monte Carlo tests shows that the distribution of the natural forests are thought of Gaussian distribution as above stated.

BESTANDESENTWICKLUNG ALS STOCHASTISCHER PROZESS

Tasiti Suzuki

Zusammenfassung

Den Entwicklungszustand von Waldbeständen kann man durch deren Durchmesserverteilung $\phi(\tau,y)$ zur Zeit τ zum Ausdruck bringen. Das Ziel dieser Arbeit ist es eine Gleichung für $\phi(\tau,y)$ abzuleiten. Nach einigen Überlegungen erhielten wir eine Art Kolmogorov-Gleichung:

$$\frac{\partial\phi(\tau,y)}{\partial\tau} = \frac{1}{2}\frac{\partial^2}{\partial y^2}[\alpha(\tau,y)\phi(\tau,y)] - \frac{\partial}{\partial y}[\beta(\tau,y)\phi(\tau,y)] - \gamma(\tau,y)\phi(\tau,y) ,$$

wobei $\alpha(\tau,y)$ die Veränderungsrate der Durchmesservarianz ist, $\beta(\tau,y)$ ist die Zuwachsrate des mittleren Durchmessers und $\gamma(\tau,y)$ ist die Wahrscheinlichkeit, daß ein Baum mit dem Durchmesser von y cm innerhalb des Zeitintervalles τ absterben wird.

Aus den in der Natur an Waldbeständen angestellten Beobachtungen können wir schließen, daß

$$\alpha(\tau,y) = const\ (M-\bar{y})^2,$$

$$\beta(\tau,y) = const\ (M-\bar{y}),$$

$$\gamma(\tau,y) = const$$

Unter Berücksichtigung dieser Koeffizienten wird die obige Gleichung zur sogenannten Bachelier - Gleichung. Sie hat die Lösung:

$$\phi_0(\tau,y) = \frac{e^{-c\tau}}{2\sqrt{a_1^2\pi(1-e^{-2k\tau})}} exp\left\{- \frac{\{y-b_1(1-e^{-k\tau})\}^2}{4a_1^2(1-e^{-2k\tau})}\right\} ,$$

wenn der Anfangszustand eine der δ-Funktion ähnliche Verteilung aufweist. Die Simulation mit Hilfe von Monte-Carlo-Tests ergibt, daß die Verteilung in natürlichen Beständen als Gauß - Verteilung gedacht wird, wie oben dargelegt wurde.

Übersetzt von J. Pollanschütz

LE DÉVELOPPEMENT DES FORÊTS EN TANT QUE PROCESSUS STOCHASIQUE

Tasiti Suzuki

Résumé

On peut interpréter l'état de développement des forêts à partir de la distribution des diamètres $\phi(\tau,y)$ au temps τ. Le but de cet exposé est de déduire une formule pour $\phi(\tau,y)$. Après quelques réflexions nous avons abouti à une espèce de formule de Komolgorov:

$$\frac{\partial\phi(\tau,y)}{\partial\tau} = \frac{1}{2}\frac{\partial^2}{\partial y^2}[\alpha(\tau,y)\phi(\tau,y)] - \frac{\partial}{\partial y}[\beta(\tau,y)\phi(\tau,y)] - \gamma(\tau,y)\phi(\tau,y) ,$$

dans laquelle $\alpha(\tau,y)$ reprèsente la modification des variations des diamètres, $\beta(\tau,y)$ l'accroissement du diamètre moyen, et $\gamma(\tau,y)$ la probabilité qu'un arbre avec le diamètre y cm mourra dans un temps τ.

Nous sommes capables de tirer des observations faites dans les forêts les conclusions suivantes:

$$\alpha(\tau,y) = const\ (M-\bar{y})^2,$$

$$\beta(\tau,y) = const\ (M-\bar{y}),$$

$$\gamma(\tau,y) = const$$

Sur la base de ces coefficients l'équation ci-dessus devient une formule dite de Bachelier se résolvant comme suit:

$$\phi_0(\tau,y) = \frac{e^{-c\tau}}{2\sqrt{a_1^2\pi(1-e^{-2k\tau})}}\exp\left(-\frac{\{y-b_1(1-e^{-k\tau})\}^2}{4a_1^2(1-e^{-2k\tau})}\right) ,$$

à la condicition que le stade initial présente une distribution identique à une fonction δ.

De la simulation à l'aide des tests de Monte-Carlo il ressort que la distribution dans des forêts naturelles est représentée comme distribution de Gauss, ainsi que nous l'avons montré plüs haut.

Traduit par L. Smidt

EL DESARROLLO DE LOS BOSQUES COMO PROCESO ESTOCÁSTICO

Tasiti Suzuki

Resumen

El estado del desarrollo de bosques puede ser expresado por medio de la distribución de sus diámetros $\phi(\tau,y)$ en el tiempo τ. El objetivo de este trabajo consiste en deducir una fórmula para $\phi(\tau,y)$. Después de algunas consideraciones obtuvimos una especie de fórmula de Kolmogorov:

$$\frac{\partial\phi(\tau,y)}{\partial\tau} = \frac{1}{2}\frac{\partial^2}{\partial y^2}[\alpha(\tau,y)\phi(\tau,y)] - \frac{\partial}{\partial y}[\beta(\tau,y)\phi(\tau,y)] - \gamma(\tau,y)\phi(\tau,y) ,$$

en la cual $\alpha(\tau,y)$ representa la modificación de las varianzas de los diámetros, $\beta(\tau,y)$ es el aumento del diámetro medio y $\gamma(\tau,y)$ es la probabilidad de que un árbol con un diámetro de y centímetros muera en el intérvalo de tiempo τ.

De las observaciones llevadas a cabo en bosques naturales podemos concluir que

$$\alpha(\tau,y) = const\ (M-\bar{y})^2,$$

$$\beta(\tau,y) = const\ (M-\bar{y}),$$

$$\gamma(\tau,y) = const$$

Teniendo en cuenta estos coeficientes la ecuación anterior se convierte en la asillamada ecuación de Bachelier, que posee la solución siguiente:

$$\phi_0(\tau,y) = \frac{e^{-c\tau}}{2\sqrt{a_1^2\pi(1-e^{-2k\tau})}}\ exp\left[-\frac{\{y-b_1(1-e^{-k\tau})\}^2}{4a_1^2(1-e^{-2k\tau})}\right] ,$$

si el estado inicial posee una distribución parecida a la función δ. De la simulación por medio de los tests de Monte-Carlo resulta que la distribución en los bosques naturales es representada como la distribución de Gauß, tal como es explica anteriormente.

Traductor : Hedwig Bloksberg